Physical bases of biophysics

Lecture 4 13. 09. 2022.

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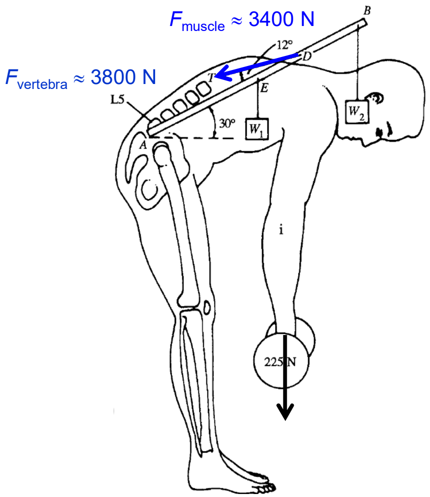
Mechanics – Dynamics and Statics



Dynamics

Newton: **motion** is **natural state**

How can we characterize the mechanical **state of motion** of an object?

v = 10 km/h =2,78 m/s

m = 20 000 kg

v = 10 km/h =2,78 m/s

m = 8 kg + ???

v = 4320 km/h =1200 m/s

m = 0,005 kg

Momentum (*p*): $p = m \cdot v \quad \left(\mathrm{kg}\frac{\mathrm{m}}{s}\right)$

vector

We usually use the letter *p* (or *I*) to denote it (from Latin pello "push, move").

Within a closed system the momentum is **conserved** (remains constant).

Momentum characterizes **translational** movement of a body.

Dynamics

Dynamics raises a new question: What is the **cause** of the **changes** of **motion** or **shape**?

Answer: The **interaction** of the object with other objects!

Newton's 1. Law/ Law of Inertia

Momentum is **conserved** (momentum conservation)

Every object remains at rest or moves in a straight line with uniform velocity until another object will compel it to change its motion.

The puck remains at rest until a force compels it to change its state of motion.

The state of motion of the puck changes because a force acts upon it.

The puck slides until an other force compels it to stop.

Reminder: The difference between the state of rest and linear motion with constant velocity depends on the inertia system.

Interactions can be of different strengths. We need a quantity that describes the **strength of the interaction**→ „**Force**".

Force

The stronger the interaction, the faster the puck accelerates ⇒ the new quantity, force (F), must be **proportional to the acceleration**:

$$\boldsymbol{F \sim a}$$

When throwing bowling balls of different weights, we can find that if the throwing is done with the same force, the lighter ball can be accelerated better than the heavier one. To achieve the same acceleration for a heavier ball, we need to exert more force. ⇒ the new quantity, the force (F) must also be **proportional to the mass**:

$$\boldsymbol{F \sim m}$$

Force (*F*): $F = m \cdot a \quad \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2} = \mathrm{N}\right)$ — vector

and $F = \dfrac{\Delta p}{\Delta t} \quad \left(\dfrac{\mathrm{kg}\frac{\mathrm{m}}{s}}{s} = \mathrm{N}\right)$ — Newton

- The direction of the force is always the same as the direction of acceleration.

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Newton's 2. Law/ Fundamental Law of Dynamics

The **change of momentum** needs **force** (F).

If more forces act on the examined object at the same time, these forces must be added (vectorially) to obtain the ***net force***:

$$F_1 + F_2 + F_3 + \cdots = \boxed{\sum F = ma}$$

Comment:
In problems/calculations, we will only work with situations where the forces act along a straight line. This simplifies vector addition for +/– operations

Special case: **equilibrium**

$\sum F = 0$ ⇒ a = 0, so the object remains at rest (v = 0) or moves with uniform velocity (v = const.).

According to this, Newton's 1. law can be viewed as a special case of the 2. law.

Statics: net force is zero, the object is at rest.

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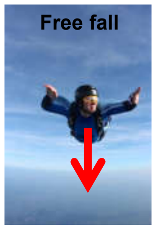
Application: Gravity

In free fall $a = g$ ⇒ so $F = m \cdot a = m \cdot g$ force is exerted on the object.

Gravity ($F_{gravity}$): $F_{gravity} = m \cdot g$

- Gravity **acts on every object** in the Earth's gravitational field, whether the body is completely in free fall or only partially, floating, or resting somewhere.



In each case, the same force of gravity exerts its effect on the objects, but the changes in motion are different! This is because other forces also act on the objects.

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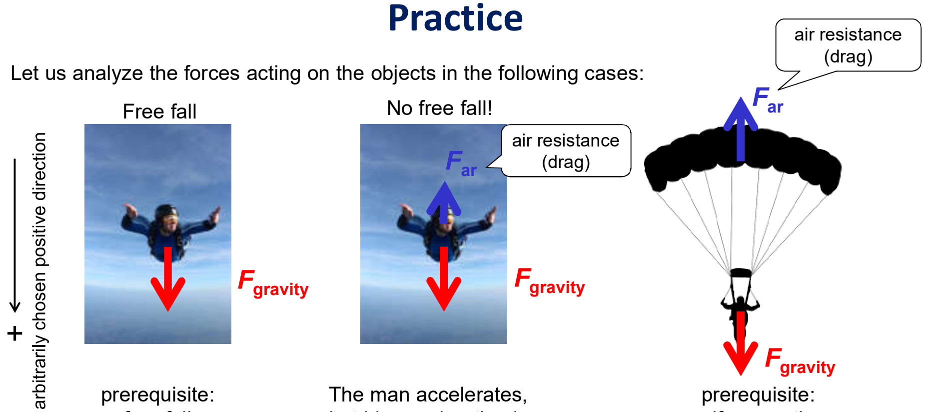
Practice

Let us analyze the forces acting on the objects in the following cases:



Free fall	No free fall!	
prerequisite: free fall	The man accelerates, but his acceleration is less than g.	prerequisite: uniform motion (v = const.)
$a = g$	$a < g$	$a = 0$
$\sum F = F_{gravity}$	$\sum F = F_{gravity} - F_{ar} = ma$	$\sum F = F_{gravity} - F_{ar} = 0$
	$F_{ar} < F_{gravity}$	$F_{gravity} = F_{ar}$

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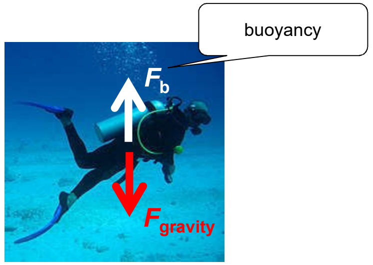
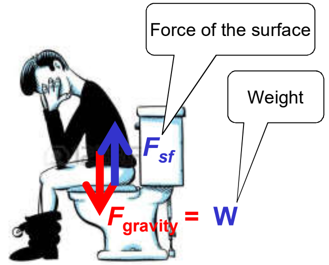
Practice

Let us analyze the forces acting on the objects in the following cases:

arbitrarily chosen positive direction ↓ +





prerequisite:
floating
(v = 0)

v = 0

$a = 0$

$a = 0$

$$\sum F = F_{gravity} - F_b = 0$$

$$\sum F = W - F_{sf} = 0$$

$$F_{gravity} = F_b$$

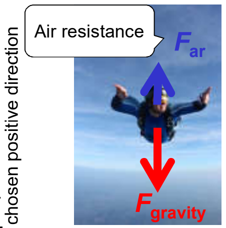
$$F_{gravity} = F_{sf} = W = mg$$

Problem

1. problem: Calulate the acceleration of the man if m = 80 kg and F_{ar} = 720 N.

No free fall!

arbitrarily chosen positive direction ↓ +



The man accelerates, but his acceleration is less than g.

2. problem: The man (m = 80 kg) falls with the acceleration of a= 2,5 m/s^2. How big is the air resistance?

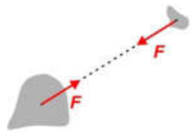
$a < g$

$$\sum F = F_{gravity} - F_{ar} = ma$$

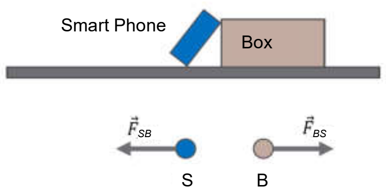
$$F_{ar} < F_{gravity}$$

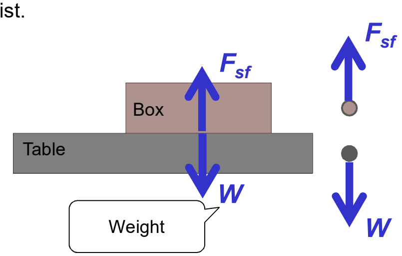
Newton's 3. Law / Law of Equal Action and Reaction

- When two bodies **interact** with each other, they **both exert a force** on each another.
- The **magnitudes of the forces** exerted on each other are the **same**, but they **point in opposite** directions. $F = -F_{versus}$



- The **forces** thus always **act in pairs**, forming force-counterforce (action-reaction) pairs. A single force cannot exist.
- Forces are always directed to **contrary parts**.



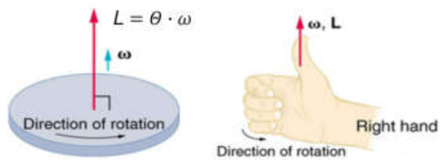


In equilibrium: $W = mg$

Newton's laws for rotation

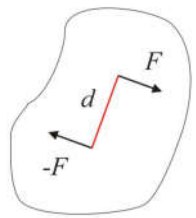
How can we characterise the **state of motion** of a **rotating object**?

Angular momentum (L): $L = \Theta \cdot \omega$ $\left(\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}}\right)$ vector



I. **$\Theta \cdot \omega$ = constant** (conservation of angular momentum) (see: rotating skater)

II. The change of angular momentum needs **torque** (M): $\frac{\Delta\Theta\omega}{\Delta t} = M$



$F_{resultant} = 0$

$M = Fd$

Equilibrium, if $F_{resultant}$ = 0 **and** $M_{resultant}$ = 0 is simultaneously fulfilled.

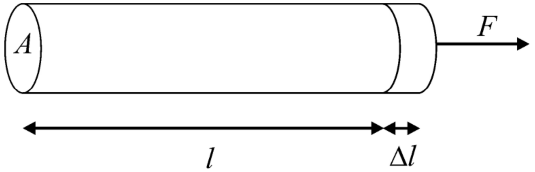
Then: $m \cdot v$ = constant and $\Theta \cdot \omega$ = constant

Change of shape

Force may result in **deformation**.

Simplest deformation is the **elongation**.

tensile strain: $\Delta l / l$



Hooke's law

$$F = AE\frac{\Delta l}{l}$$

$$\frac{F}{A} = E\frac{\Delta l}{l}$$

F/A is the **stress** (**tensile stress**) (σ [Pa]),
but it could be **compressive stress** or **pressure** (p [Pa])
Coefficient: **Young's modulus** (E[Pa])

e.g.: collagen fiber 0,3–2,5 GPa, bone 10–20 GPa

More general - **compressive stress**: $\Delta p = -K\frac{\Delta V}{V}$

K is the bulk modulus,
$1/K = \kappa$ is the compressibility (pl. κ_{steel} = 0,006 GPa-1)

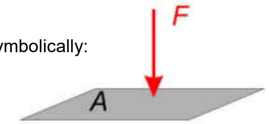
Pressure

- The deformation of an object depends not only on the force acting on it, but also on the surface on which the force is applied.
- Power alone is not always enough to describe the interaction. We need a **new quantity that also takes the surface into account**. → „pressure".

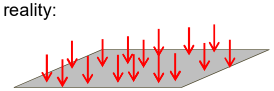
pressure (p): $p = \frac{F}{A} \quad \left(\frac{\text{N}}{\text{m}^2} = \text{Pa}\right)$

Pascal

symbolically:



in reality:



(even distribution)

Other commonly used units:
bar (bar) = 100 kPa; atmosphere (atm) = 101, 325 kPa;
millimetre of mercury (mmHg) = 133,3 Pa

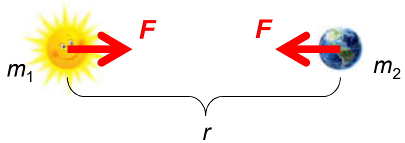
When this description is not enough

- At speeds close to the speed of light
  → special theory of relativity
- For objects of atomic size
  → quantum mechanics
- Non-inertial systems (e.g. accelerating airplanes) require other forms of equations

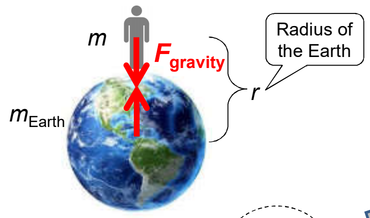
Special types of forces and their laws - gravitational force and the law of universal gravition

$$F = G\frac{m_1 \cdot m_2}{r^2}$$

Gravitational constant

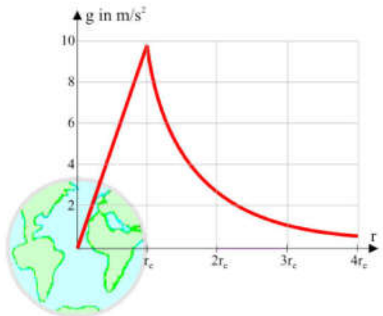


Gravity on the Earth:



$$F_{gravity} = G\frac{m_{Earth}}{r^2}m = mg$$

g



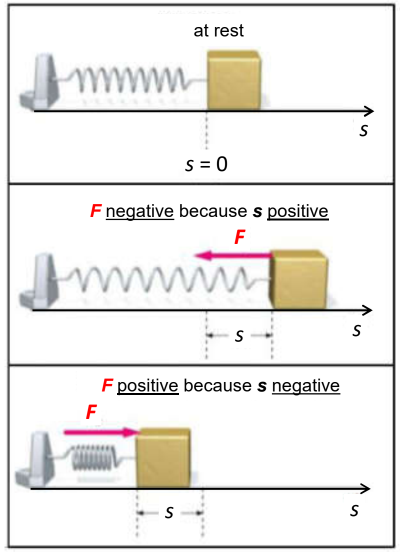
Special types of forces and their laws – force of a spring and Hooke's law

The result of a force (interaction) can not only be a change of motion but also a change of shape (deformation).

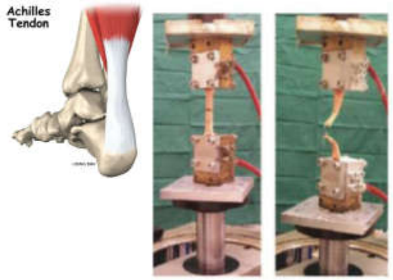
$$F = -k \cdot s$$

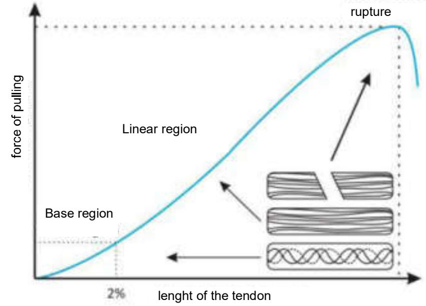
spring constant
(N/m)
Influnced by the properties of the spring (material, geometry).

- This force is also called **restoring force**.



Biomechanics of tendons and ligaments

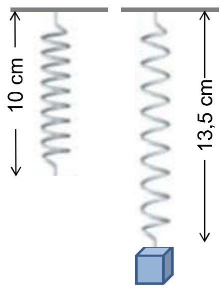




Hooke's law applies approximately to the Achilles tendon, which can therefore be modeled with a spring.

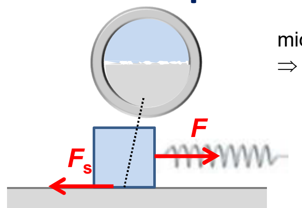
A force of 1200 N is required for 2% elongation of the Achilles tendon of the lenght of 10 cm. Calculate the spring constant of the tendon!

Problem



The spring constant of the spring shown in the figure is 500 N/m. Calculate the weight of the object placed on it!

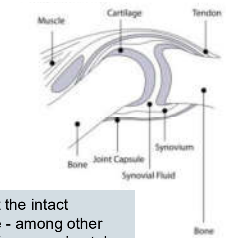
Special types of forces – Friction



microscopic contact surface – molecular forces of attraction
⇒ friction

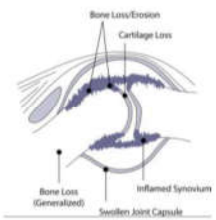
The constant spring force of 20 N is applied and the object glides evenly. What is the force of friction?

healthy joint



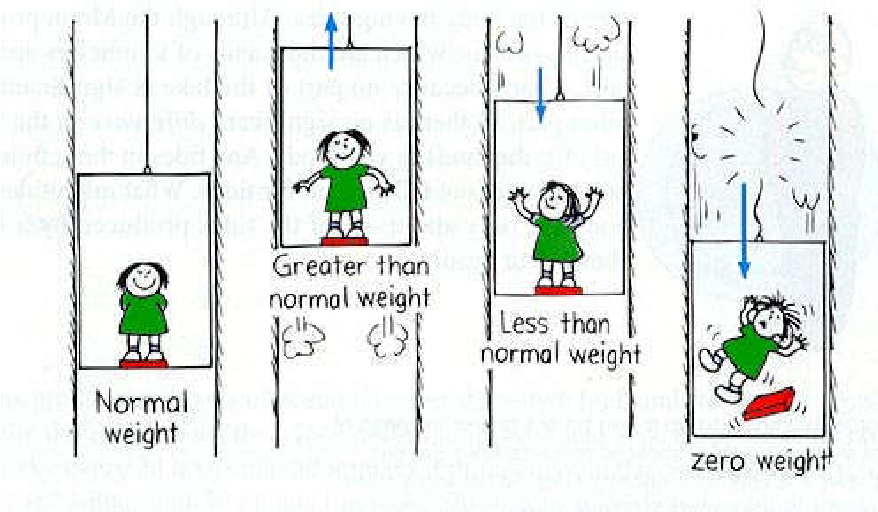
In a healthy joint the intact cartilage surface - among other factors - allows for approximately friction-free movement.

joint in rheumatoid arthritis



Injury to the cartilage surface e.g. in rheumatoid arthritis, it increases friction in the joint.

Weight



Homework: Chapter 4