

Mathematical and Physical Basis of Medical Biophysics

Chapter 5 Mechanics –Work and Energy

Energy-Work

Work and energy describes the interactions between objects, can be applied more widely than force (e.g. for thermal or chemical interaction)

Energy:

- Describes the state of an object or a system
- Ability of a system to perform work

Work: is done on an object when you transfer energy to that

Different kinds of energies

- **Kinetic energy**
- **Potential energy**
- Internal energy
- Chemical energy
- Nuclear energy
- Electrical energy

... they can be converted into each other

Work

For mechanical interactions:

- Object accelerated by force
- Lifting up an object
- Extending a spring

$$W = F \cdot s$$

(if force and displacement have the same direction)

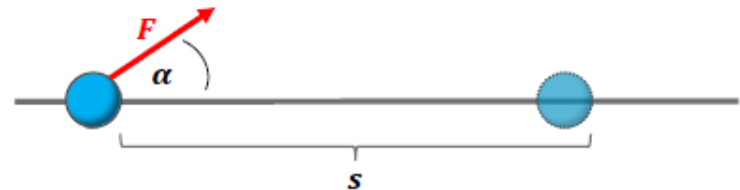


W: Work [N · m=**Joule**=J] (scalar)

F: force [N=Newton= $\frac{kg \cdot m}{s^2}$]

s: displacement of the object [m]

$$W = F \cdot s \cdot \cos \alpha$$



α : angle between the force and the displacement

Power

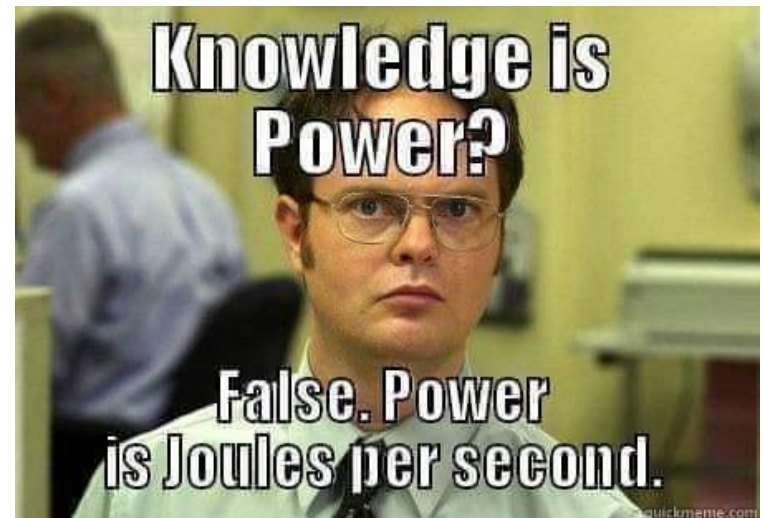
- Rate of doing work

$$P = W/t$$

P: power [J/s=**watt=W**]

W: Work [N · m=Joule=J]

t: time [s]



Energy and work have the same unit: **J**

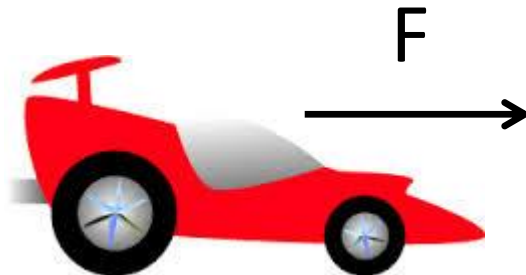
Unit conversions

	Joule	Electronvolt	Calorie
1 J=	1	$6.25 \cdot 10^{18}$	0.239
1 eV=	$1.6 \cdot 10^{-19}$	1	not relevant
1 cal=	4.19	not relevant	1

Problem V/2

A car ($m=1,2\text{ t}$) is uniformly accelerating from rest for 12 seconds to reach a velocity of 100 km/h.

- a) Calculate the force necessary for this acceleration.
- b) Calculate the distance run by the car during acceleration.
- c) Calculate the work done by the accelerating force.
- d) Calculate the average power of the car.
- e) Calculate the kinetic energy of the car at the end of the acceleration.



Types of mechanical energy

- Kinetic energy

(accelerating an object)

- Gravitational potential energy

(lifting up an object)

- Elastic energy

(extending a spring)

Kinetic energy

- Related to motion

$$E_{kin} = \frac{1}{2} \cdot m \cdot v^2$$

E_{kin} : kinetic energy [J=Joule]

m : mass [kg]

v : velocity [m/s]

Work done during acceleration: $W = F \cdot s = m \frac{v}{t} \cdot \frac{v}{2} t = \frac{1}{2} m \cdot v^2.$

Potential energy

- Results from a position or configuration

Depending on the force field it can be:

- **Gravitational**
- Magnetic (later)
- Electric (later)

Elastic energy: configuration dependent potential energy

Gravitational potential energy

- A capacity for doing work as a result of the object position in a gravitational field

$$E_{pot} = m \cdot g \cdot h$$

E_{pot} : potential energy [J=Joule]

m : mass[kg]

g : gravitational acceleration=9.81 [m/s²]

h : height above a reference level [m]



Karim Kouz, Biophysik WS2015/2016

Work done during elevation:

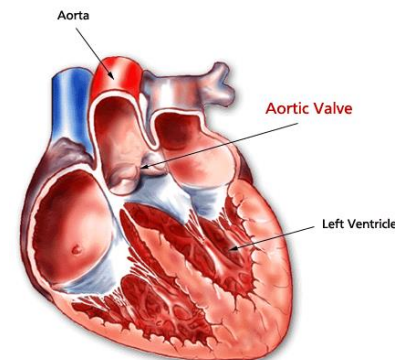
$$W = F \cdot s = mgh$$

Problem V/4

The left ventricle pump about 70 g of blood in a contraction into the aorta. This amount of blood reaches the aortic arch that is located approximately 15 cm above the ventricle and has a flow velocity of 30 cm/s.

Calculate:

- a) the work needed to lift the blood,
- b) the work needed to accelerate the blood,
- c) the power of the left ventricle during a contraction that lasts for 0,2 s!



Problem V/5

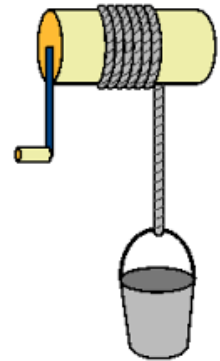
Someone is pulling a bucket full of water ($m=12$ kg, including the mass of 10 litres of water in it) to the top of an 8 m deep well, with uniform velocity of 50 cm/s. Calculate

a) the force acting on the bucket

b) the work done

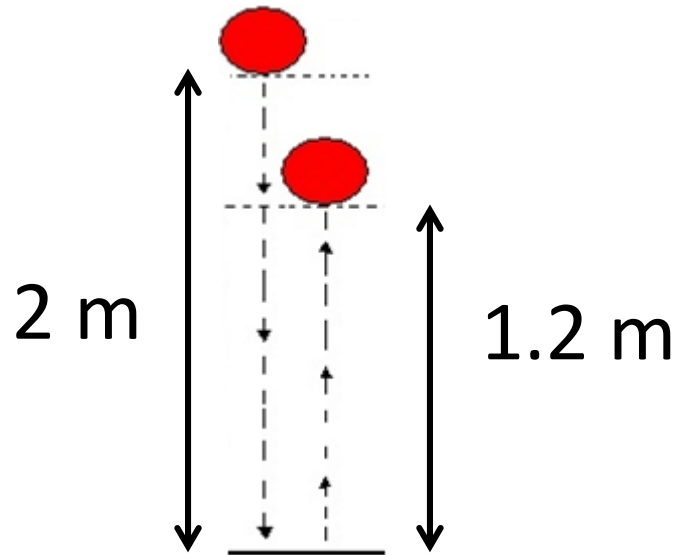
c) the power

d) How many kcal of energy equals the work of a man who is lifting up a total of $4,8 \text{ m}^3$ water from the well during one day?

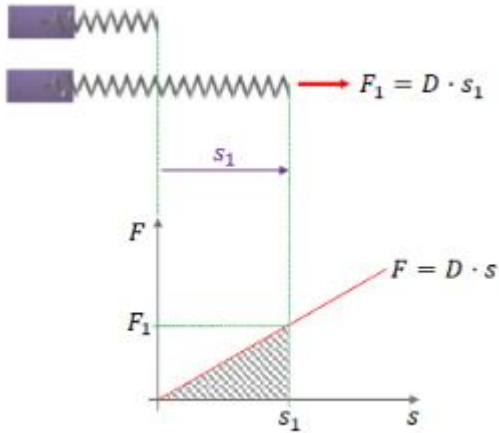


Problem V/9

A ball ($m=0,8$ kg) hits the floor from a height of 2 m and bounces back to the height of 1,2 m. Calculate the amount of energy lost due to air drag and the collision with the ground.



Elastic energy



$$E_{elastic} = \frac{1}{2} \cdot k \cdot s^2$$

$E_{elastic}$: elastic potential energy [J=Joule]

k : spring constant [N/m]

s : deformation of an elastic object [m]

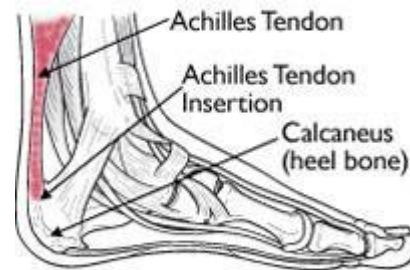
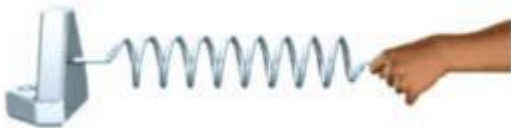


Work done during the stretch:

$$W = F \cdot s = \frac{1}{2} k s \cdot s = \frac{1}{2} k s^2$$

Problem V/7

Calculate the amount of energy stored in the Achilles tendon with a spring constant of $3 \cdot 10^5 \text{ N/m}$ that is extended by 2mm.



Conservation of energy for mechanics

- Total amount of energy in an isolated system remains constant
- Isolated system: neither matter nor energy can pass

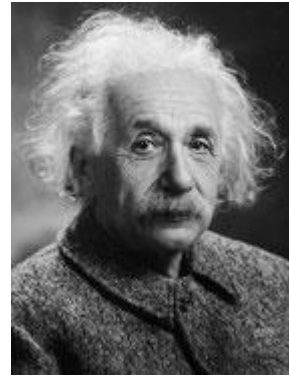
$$E_{kin} + E_{pot} + E_{elastic} = \textit{constant}$$

$$\frac{1}{2} \cdot m \cdot v^2 + m \cdot g \cdot h + \frac{1}{2} \cdot k \cdot s^2 = \textit{constant}$$

**"Energy can neither be created
nor destroyed'.
Émilie du Châtelet**



Mass-energy equivalence



- Every object of mass m , has a rest energy:

$$E = mc^2$$

E : rest energy[J=Joule]

m : mass [kg]

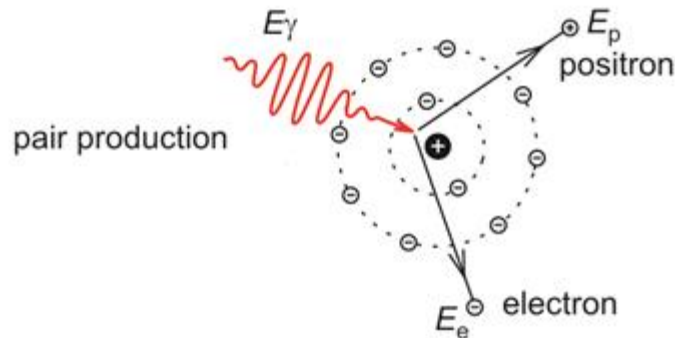
c : speed of light in vacuum $=3 \cdot 10^8$ [m/s]

Mass and energy can be transferred into each other.

e.g. PET

Problem V/10

Calculate the rest energy of an electron ($m_e = 9,11 \cdot 10^{-31}$ kg)! Convert your result to eV unit !

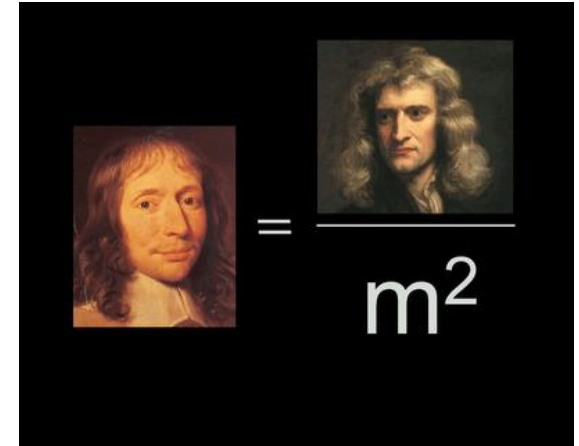
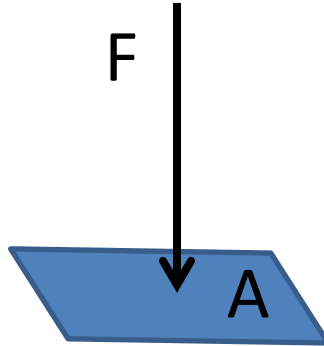


Chapter 6

Mechanics –Pressure

Pressure

$$p = \frac{F}{A}$$



p: pressure [N/m²=Pa=Pascal]

F: force applied perpendicular to the surface [N]

A: surface of an object [m²]

Pressing something with palm or finger cause different deformation

Units of pressure

SI unit: pascal ($\text{Pa} = \text{N/m}^2$)

Other units:

$$1 \text{ bar} = 10^5 \text{ Pa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 1.01 \cdot 10^5 \text{ Pa} = 101 \text{ kPa} = 1.01 \text{ bar} = 760 \text{ mmHg}$$

$$1 \text{ mmHg} = 1 \text{ torr} = 133 \text{ Pa} = 0.133 \text{ kPa}$$

Problem VI/2

Masticatory forces of a human can reach up to 100 N (for crocodiles it is 1000 N!). When someone bites on a bone chip in the burger or on the seed of a fruit, this force is concentrated on a surface area of 1 mm^2 . Calculate the pressure!



Problem VI/3

- a) Calculate the pressure that a 70 kg standing man exerts on the floor. (The total surface of the two soles is 200 cm^2)
- b) Calculate the pressure this man exerts on the ice surface during skating! (The total surface of the blades is 4 cm^2)



Density

- Pressure in gases and fluids depends on density

$$\rho = \frac{m}{V}$$

if it is homogenous

ρ : density [kg/m³]

m: mass [kg]

V: volume [m³]

Density of materials

material	ρ (g/cm ³)
air (at 0°C and 101 kPa)	0,00129
water (at 4°C and 101 kPa)	1
water (at 100°C and 101 kPa)	0,958
ice	0,92
aluminium	2,7
mercury	13,6
gold	19,3
human body (averaged)	1,04

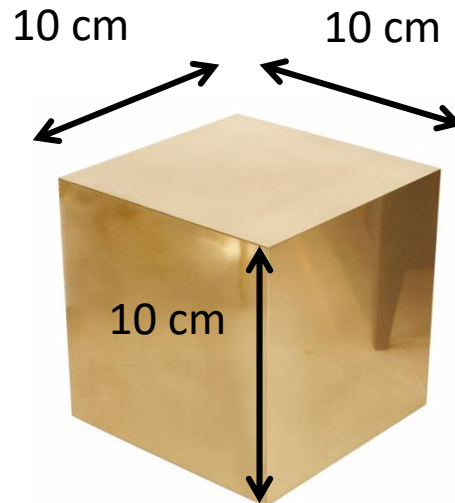
$$1\text{g/cm}^3 = 1\text{kg/dm}^3 = 1000\text{ kg/m}^3$$

$$\text{ml} = \text{cm}^3$$

$$\text{liter} = \text{dm}^3$$

Problem VI/5

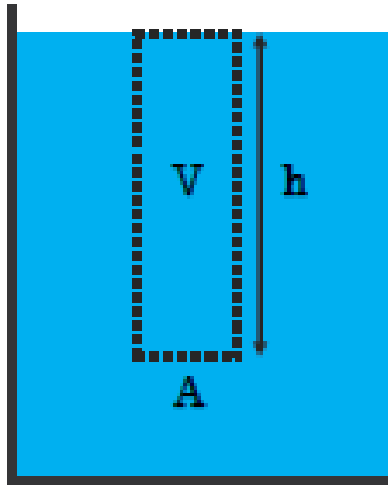
- a) Calculate the mass of a gold cube with the width of 10 cm!
- b) Calculate the pressure exerted by this cube on a horizontal shelf, that holds it!



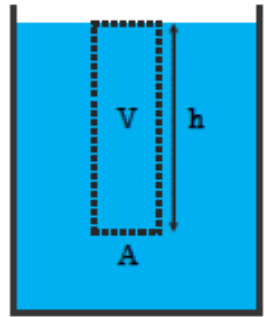
Hydrostatic pressure

Spontaneously arising in fluids and gases caused by the gravitational field of Earth

The ,body' (gas or fluid) with height , h ' is pressing the underneath surface , A ' with its weight.



Hydrostatic pressure



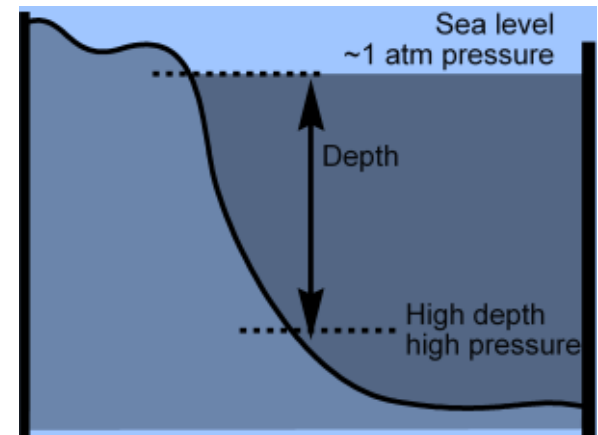
$$W = m \cdot g = \rho \cdot V \cdot g = \rho \cdot A \cdot h \cdot g$$

means weight, not work

$$p = \frac{F}{A} = \frac{W}{A} = \frac{\rho \cdot A \cdot h \cdot g}{A} = \rho \cdot h \cdot g$$

means pressure, not power

Pressure (in fluids and gases) is directly proportional to depth



The hydrostatic paradox

At the bottom of which container we have the highest pressure?

$$p = \rho \cdot g \cdot h$$



Hydrostatic pressure depends on:

- density
- height of fluid

Does NOT depend on the shape of the container

Problem VI/7

Calculate the pressure (that is total pressure!) at 1 km below the sea surface! Let us assume that the density of sea water is $1,08 \cdot 10^3 \text{ kg/m}^3$ at every depth.

