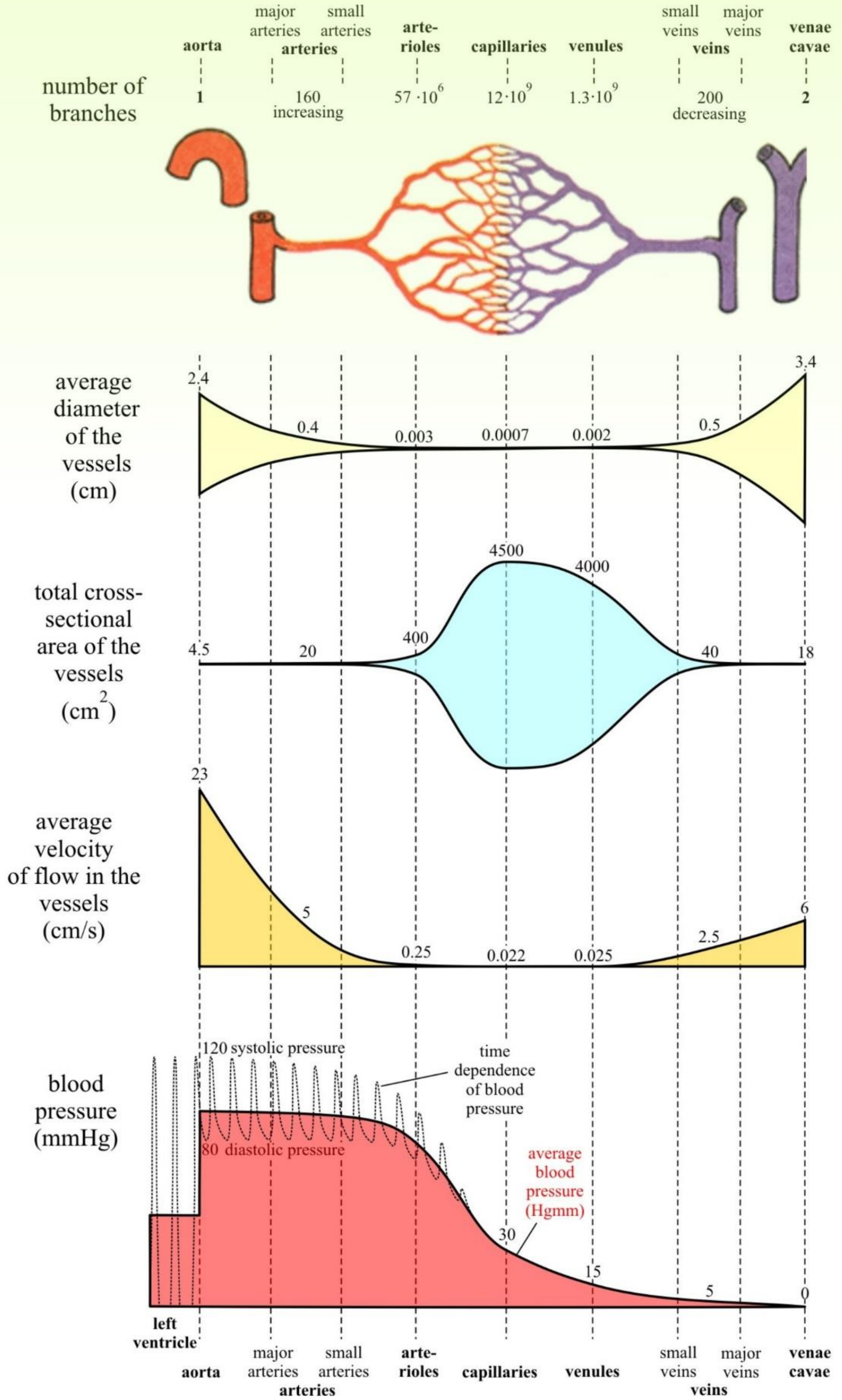
FLOW

FLOW OF FLUIDS



SUMMARY

VISCOSITY (η): A coefficient that characterizes the internal friction of fluids and gases. It is the ratio between the shear stress and the velocity gradient. Its value is greater for fluids that flow less easily. It depends on the material and its temperature. The unit of viscosity is Pa·s.

NEWTONIAN FLUID: The fluid is Newtonian if its viscosity does not depend on the shear stress or spatial velocity change. Water is a Newtonian fluid.

STATIONARY FLOW: Parameters of the flow (e.g., velocity, pressure) are temporally invariant, and constant at any point of the flow.

LAMINAR FLOW: At small flow velocities, fluid flows in layers (lamina) that do not mix. The directions of the velocities of the particles composing the fluid are parallel with that of the flow.

TURBULENT FLOW: Above a critical flow velocity, the fluid layers mix, thereby forming complex swirls and eddies. The directions of particle velocities have components orthogonal and parallel to the flow velocity.

HAGEN-POISEUILLE LAW: Equation that describes stationary laminar flow of Newtonian fluids in rigid tubes ($I_V = \Delta p / R_{flow}$). The volumetric flow rate ($I_V = \Delta V / \Delta t$) is directly proportional to the pressure drop (Δp) that maintains the flow. The proportionality factor is the reciprocal value of the resistance to flow (R_{flow}).

RESISTANCE TO FLOW (or resistance of the tube): Directly proportional to the length of the tube (l), viscosity of the fluid (η) and inversely proportional to the fourth power of the radius of the tube (r), thus the square of the cross-sectional area (A).

$$R_{flow} = R_{tube} = \frac{8 \cdot l \cdot \eta}{\pi \cdot r^4} \sim \frac{l \cdot \eta}{A^2}.$$

Stephen Hales, English doctor and inventor, was the first to measure the blood pressure of a horse in 1733. The blood of the horse rose 9 feet high in a glass tube placed inside the brass cannula. Unfortunately, the horse did not survive the experiment.

Blood, the most important body fluid, flows in the vascular system, a network of branching tubes (blood vessels) of different cross-section and length. The pressure difference necessary for maintaining blood flow is generated by the heart. In this lab we will examine the relationship between volumetric flow rate and pressure drop, we will verify the relationship between the flow rate and the radius of the tube and based on our measurements we will calculate the viscosity of water.

Further readings: Damjanovich-Fidy-Szöllősi: III /1.

THEORETICAL OVERVIEW

FLOW OF FLUIDS IN TUBES

he **volumetric flow rate (I_V)** is an important parameter of fluid flow, defined as the volume (ΔV) of the fluid that flows through a cross-section of the tube in time Δt. Its unit is m^3/s:

$$I_V = \frac{\Delta V}{\Delta t}\text{ , or in other form: } I_V = \frac{A \cdot \overline{\Delta l}}{\Delta t} = A \cdot \bar{v}, \qquad (1)$$

where A is the cross-sectional area of the tube, $\overline{\Delta l}$ is the average displacement of the fluid, and $\bar{v}$ is the average speed at the given cross-section.

flow intensity
Volumenstromstärke
térfogati áramerősség

In case of **stationary flow**, the parameters of the flow (e.g., speed, pressure) do not change in time, but remain **constant** at any point of the flow.

Continuity equation states that in stationary flow of an incompressible fluid (it is not always true for gases, see comment), the volumetric flow rate is the same at any point along the tube:

In case of normally breathing in and out, air is not compressed, thus in this sense can be considered incompressible. Small pressure changes do not change the density of air significantly. However, for example, during cough, as a consequence of larger pressure changes the air is compressed.

$$I_V = A_1 \cdot \overline{v_1} = A_2 \cdot \overline{v_2} = const. \qquad (2)$$

According to this equation, if a fluid flows in a tube that varies in cross-sectional area, **the same volume flows through any cross section of the tube in the same time** (Fig. 1). This implies that in the narrower section ($A_2 < A_1$) the fluid flows faster ($\overline{v_2} > \overline{v_1}$) than in the larger cross section.

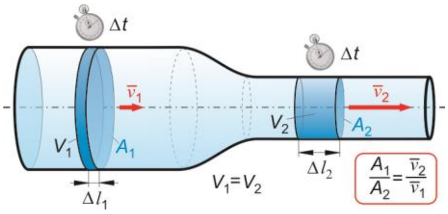


***Fig. 1.** Continuity equation: the same volume flows through any cross section of the tube in the same time.*

Viscosity (η) is a coefficient characterizing the internal friction of fluids and gases. Its value is greater for viscous fluids flowing less easily. For example, honey is more viscous than blood, and blood is more viscous than water (Fig. 2). The unit of viscosity is Pa·s. If the coefficient of viscosity does not depend on the spatial change of the speed (i.e., viscosity is constant across the cross-section of the tube) or the shear stress that developed by the flow, then the fluid is called **Newtonian**.

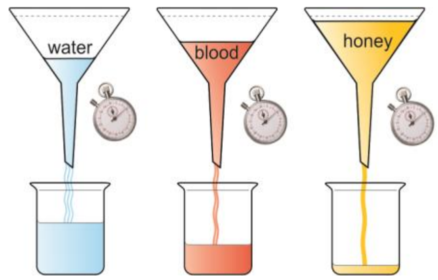


***Fig. 2**. Fluids of different viscosity flow through the openings of the same cross-section with different velocity*

material	viscosity (mPa·s)
air	0.02
ether	0.23
water	1
mercury	1.55
blood	2 - 4.5
glycerol	1500

***Table 1.** Viscosity of various materials at 20 °C temperature and 101 kPa pressure.*

viscosity, internal friction
Viskosität
viszkozitás, belső súrlódási együttható

newtonian fluid
newtonsche Flüssigkeit
newtoni folyadék

The flow of a fluid can be visualized by **streamlines**. Streamlines become visible if the fluid is colored with a bright dye at several points along the tube's cross-section. Streamlines highlight the local direction of flow (Fig. 3). At low speeds, the colored fluid does not mix with the transparent one, thus the fluid flows as if individual fluid layers slid past one another. This type of flow is called **laminar**. If flow velocity exceeds a critical value, then the streamlines highlight a complex, swirling path particularly in the center of the tube (where flow velocity is maximal). This type of flow is called **turbulent.**

laminar flow; turbulent flow
laminare Strömung; turbulente Strömung
lamináris áramlás; turbulens áramlás

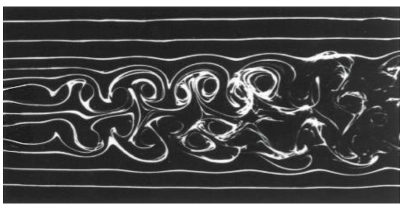
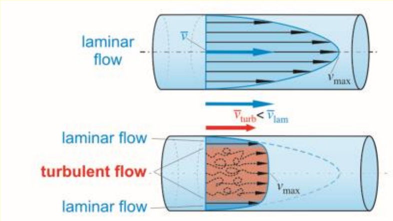


***Fig. 3.** Laminar and turbulent flow. Right next to the tube walls (at low speeds) the flow is laminar, but in the center (at high speeds) it is turbulent.*

In case of **laminar flow,** the speed of the fluid layers, thus that of the fluid cylinders sliding telescopically in each other, changes due to internal friction, because each layer of fluid exerts a drag force on adjacent layers. The speed of flow is highest in the center of the tube and decreases parabolically upon approaching the wall, where the fluid sticks to it (velocity profile is of paraboloid shape).



Laminar and **turbulent flow** may occur simultaneously as well (see Fig. 3). In this case, streamlines in the central part of the tube show an irregularly swirling pattern, while next to the walls laminar flow is maintained.
The following criteria are used to determine which type of flow is dominant.
Laminar flow in the tube becomes turbulent if the dimensionless number calculated from the parameters of the flow, called the Reynolds number (Re),

$$Re = \frac{\bar{v} \cdot \rho \cdot r}{\eta},$$

exceeds a critical value: $Re_{crit} = 1160$. From this we can calculate the corresponding critical flow velocity:

$$\bar{v}_{crit} = 1160 \frac{\eta}{\rho \cdot r}.$$

The flow conditions in the blood vessels (v,r) and the physical parameters of blood (ρ,η) are such, that **normally the blood flow is laminar**. Turbulent flow is disadvantageous physiologically, because the average velocity of blood ($\bar{v}_{turb} < \bar{v}_{lam}$) and thus the volumetric flow rate decreases significantly. Although it can be compensated by increasing the blood pressure, it also increases the work of the heart.

Let us examine the flow conditions in the aorta, where:
$r = 0.0125$ m, $\rho = 1050$ kg/m^3, $\eta = 4 \cdot 10^{-3}$ Pas.
We get the critical velocity from the equation above:

$$\bar{v}_{crit} = 1160 \frac{4 \cdot 10^{-3}}{1050 \cdot 0.0125} \cong 0.35 \frac{\mathrm{m}}{\mathrm{s}}.$$

The actual average velocity of flow in the aorta is:

$$\bar{v} = 0.24 \frac{\mathrm{m}}{\mathrm{s}} < v_{crit} = 0.35 \frac{\mathrm{m}}{\mathrm{s}},$$

In some clinical conditions, the radius (r) of the aorta is decreased, thus the cross-section (A) is smaller. As a result, the value of critical velocity ($\bar{v}_{crit}$) increases, but the flow velocity increases even more rapidly according to the equation of continuity (2). Thus it can easily exceed the critical velocity, and the flow may become turbulent.

Conventional blood-pressure measurement is based on listening in the cubital fossa, a sound that is produced by turbulent flow (Korotkoff sounds).

The pathological decrease of the viscosity of blood (η) may cause turbulent flow as well.

Because laminar flow is relevant under physiological conditions, in the following we will concentrate on this type of flow and mention the effects of turbulence only occasionally.

The equation that describes stationary laminar flow of Newtonian fluids in rigid tubes is the **Hagen-Poiseuille law**:

$$I_V = \frac{\Delta V}{\Delta t} = \frac{\pi}{8} \cdot \frac{r^4}{\eta} \cdot \frac{p_1 - p_2}{l}. \qquad (3)$$

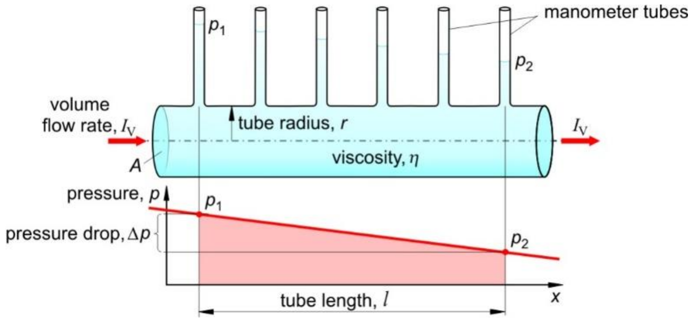


Fig. 4. *Laminar flow of a viscous fluid or gas is maintained by a pressure difference.*

In order to maintain the volumetric flow rate, I_V, in a circular tube of radius r and length l of the fluid or gas with a viscosity of η, a static pressure difference of $\Delta p = p_1 - p_2$ is required (Fig. 4). Upon rearranging equation (3) we obtain that the **volumetric flow rate is directly proportional to the pressure drop**:

$$I_V = \frac{\Delta p}{\left(\frac{8 \cdot l \cdot \eta}{\pi \cdot r^4}\right)}, \text{ that is } I_V = \frac{\Delta p}{R_{flow}}, \qquad (4)$$

where the $1/R_{flow}$ is the coefficient of proportionality. By rearranging the Hagen-Poiseuille equation, we can see which parameters of the tube define its flow resistance:

$$R_{flow} = \frac{\Delta p}{I_V} = \frac{8 \cdot l \cdot \eta}{\pi \cdot r^4} \cdot \frac{\pi}{\pi} = \frac{8 \cdot l \cdot \eta \cdot \pi}{\pi^2 \cdot r^4} = \frac{8 \cdot l \cdot \eta \cdot \pi}{(\pi \cdot r^2)^2} = 8 \cdot \pi \cdot \eta \frac{l}{A^2}. \qquad (5)$$

As we can see, while the flow resistance is directly proportional to the length of the tube, it is **inversely proportional to the square of the cross-sectional area (*A*).** As we can see, while the flow resistance is directly proportional to the length of the tube, it is **inversely proportional to the square of its cross-sectional area (*A*)**. Therefore, even a little change in the radius of the blood vessels changes significantly the resistance to flow, and hence the volumetric flow rate (Fig. 5).

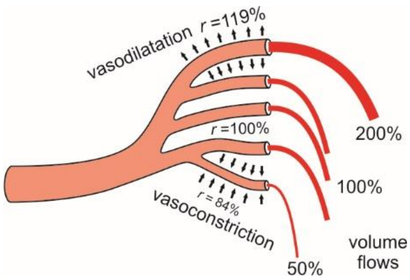


Fig. 5. *Even a little change in the radius of the vessels induces significant change in the volumetric flow rate.*

FLOW OF BLOOD IN THE CIRCULATORY SYSTEM

The heart is the motor of the cardiovascular system. By generating pressure via muscle contraction, it pumps blood into the vascular system composed of serially connected vessels of different types (arteries, arterioles, capillaries, venules and veins), hence transports blood through the organism. The system of arteries carries blood away from the heart in different **types of vessels**. The **aorta** divides into **major arteries** that further divide into **small arteries**, then into **arterioles** and **capillaries**. These vessels are of increasing number of branches of smaller and smaller diameter. The venous system is arranged symmetrically, but in the opposite direction, as smaller veins converge into larger ones (of larger diameter) until they reach the heart (see cover figure).

The validity of the Hagen-Poiseuille law for the flow of blood in the circulatory system is limited. This law is valid only for the conditions mentioned earlier (Newtonian fluid, stationary laminar flow, rigid tube). However, the flow of blood differs from these because of some important features. Firstly, **blood flow is pulsatile**, especially in the aorta and somewhat less in the arteries, thus it is not stationary. Second, **blood is not a Newtonian fluid**. Third, **the walls of the blood vessels are** not rigid, but **elastic**. During systole the large arteries distend due to the suddenly rising pressure as the stroke volume is ejected from the ventricles. Because the rate of blood entering the arteries exceeds that leaving them, they extend thus their elastic potential energy increases. During diastole as the valves close, the pressure drops and the distended arterial walls contract while the elastic potential energy stored in them is converted into the kinetic energy of blood flow (see Fig. 6). As a result, the pulsating pressure waves produced by the heart are damped to some extent (see cover figure, bottom). As a result, the fluctuations in the volumetric flow rate are also damped and the average flow rate increases without increasing the work of the heart.

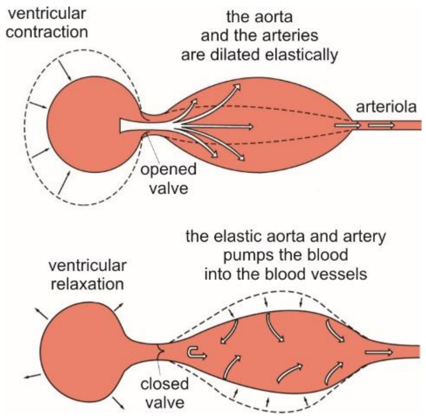
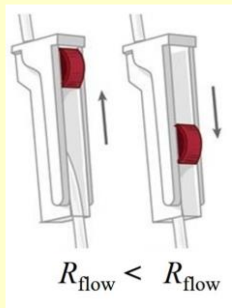


Fig. 6. *Role of the elastic arteries (The windkessel effect).*

Neural control of blood vessel radius has a dramatic effect on their resistance to flow (Fig. 5.).
Arterioles are capable of increasing their radius by 50% (vasodilatation). At the same pressure difference and viscosity the volumetric flow rate increases $1.5^4 \approx 5$ times. As a counterexample, 16 % decrease in the radius reduces blood flow by half in the given section. In this case, the original blood flow would be restored at twice as high blood pressure.

In infusion therapy the volumetric flow rate is regulated with a roller clamp placed around the tubing. When the wheel is rolled down, it compresses the elastic tubing, thus reducing its cross sectional area. This will result in the increase of flow resistance.



EXPERIMENTS

ELASTICITY OF BLOOD VESSEL WALLS IN CASE OF PULSATILE BLOOD FLOW

1. demonstration. Compare pulsatile and stationary flow in the elastic and rigid tubes with a simple mechanical model. The result speaks for itself.

DEPENDENCE OF VOLUMETRIC FLOW RATE ON THE TUBE RADIUS

2. demonstration. Using the flow model shown in Fig. 7. determine the flow rate for each tubes as we flow water through them. The flow id driven by the hydrostatic pressure of the cylindrical container. Measure the flow through time of the same volume for each tube and collect the results in the lab report.

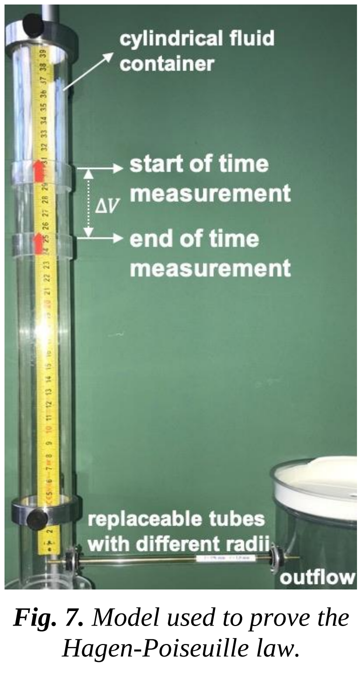


Fig. 7. *Model used to prove the Hagen-Poiseuille law.*

TASKS

1. Based on the flow through times and volumes measured with the Hagen-Poiseuille model, calculate the volumetric flow rate for each tube!
2. Plot the calculated flow rates as a function of the tube radius! Determine the mathematical function between the variables by fitting!
3. Calculate the viscosity of water using the data of the 1 mm radius tube!

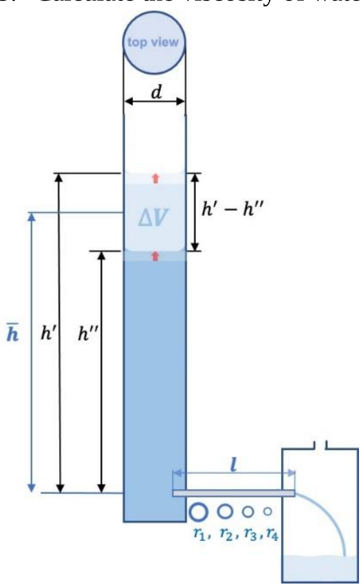


Fig. 8. *Schematics of the flow model.*

FORMULAE USED IN THE EVALUATION:

Flow through volume (ΔV):

$$\Delta V = \pi \cdot \left(\frac{d}{2}\right)^2 \cdot (h' - h'')$$

Average height of fluid column ($\bar{h}$):

$$\bar{h} = \frac{(h' + h'')}{2}$$

Average pressure drop (Δp_{avg}):

$$\Delta p_{avg} = \rho \cdot g \cdot \bar{h}$$

Viscosity (η):

$$\eta = \frac{\pi \cdot r^4 \cdot \Delta p_{avg}}{8 \cdot I_V \cdot l}$$

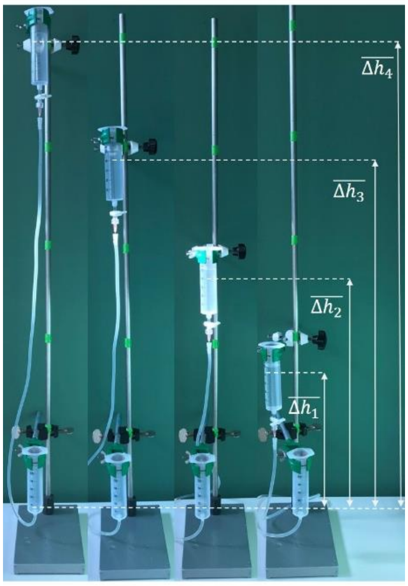


Fig. 9. *Measurements with the infusion model at various height differences.*

The total resistance of a flow system can be calculated as the ratio of the pressure drop measured between the termini of the system and the volumetric flow rate.

$$R_{\text{flow}} = \frac{p_1 - p_2}{I_{\text{V}}}$$

In case of the human cardiovascular system two circuits and the pressure generating heart are connected in series. Therefore, we can calculate the so called peripheral resistance for both circuits. In case of the systemic circulation it is called **total peripheral resistance (TPR)**:

$$\text{TPR} = \frac{p_{\text{aorta}} - p_{\text{right atrium}}}{\text{cardiac output}}$$

At rest, the mean pressure in the aorta is about 93 mm Hg and the pressure in the right atrium is 2 mm Hg. If we assume a cardiac output of 5,5 l/min, then:

$$\text{TPR} = \frac{93\ \text{mm Hg} - 2\ \text{mm Hg}}{5{,}5\ \text{l/min}} = 16{,}5\ \frac{\text{mm Hg}\cdot\text{min}}{\text{l}}$$

DEPENDENCE OF VOLUMETRIC FLOW RATE ON PRESSURE DROP

With the infusion model shown in Fig. 9. determine the relationship between the volumetric flow rate and pressure drop of flow system. The model composed of two syringes connected by a silicone tubing. The top syringe is connected to the tubing through an 18 gauge (G18) needle and a valve. You will measure the flow through time of given a volume (ΔV) at various average height differences.

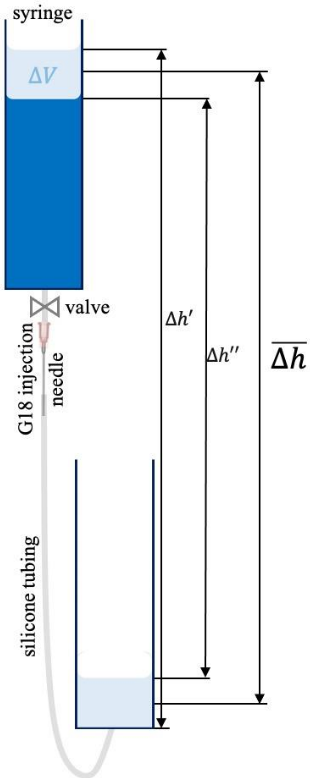


Fig. 10. *Schematics of the infusion model.*

TASKS

1. Measure the flow through times of $\Delta V = 20$ ml water at four different heights! Collect the results in your lab report!
2. Calculate the average hydrostatic pressure differences and the volumetric flow rate for all four measurements!
3. Plot the volumetric flow rate as a function of the average hydrostatic pressure differences!
4. Determine the mathematical function between the variables by fitting and calculate the flow resistance of the model!