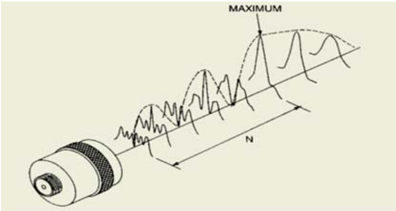
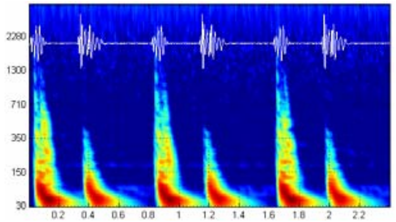
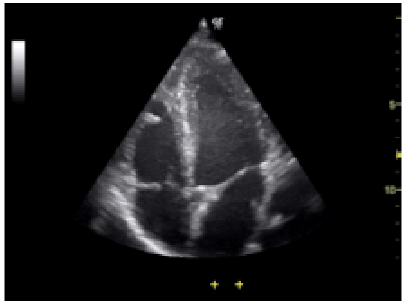
Physics of ultrasonography







KAD 2022.02.16

question in the cellar of a pub: how much wine is in the barrel?
medical question: how much air is in the lungs?

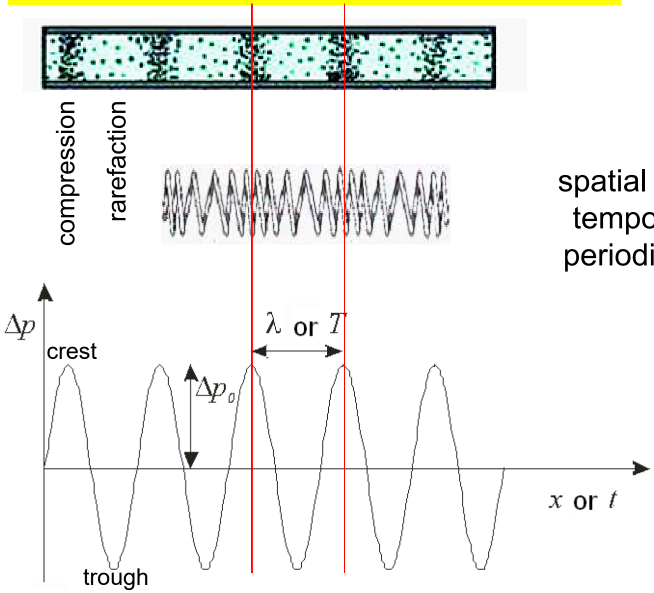
Auenbrugger (son of an innkeeper in Graz, 1761): **percussion** for testing the air content of hollow organs

Sound: mechanical wave (model)

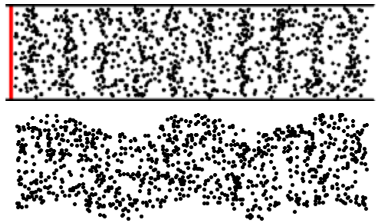
whistle

spring

function



spatial and temporal periodicity



longitudinal wave
(in the interior of liquids and gases only this type)

transverse wave

hydrostatic pressure | pressure change, sound pressure

$p_{total} = p_{hydrostat} + \Delta p$

pressure DC + AC

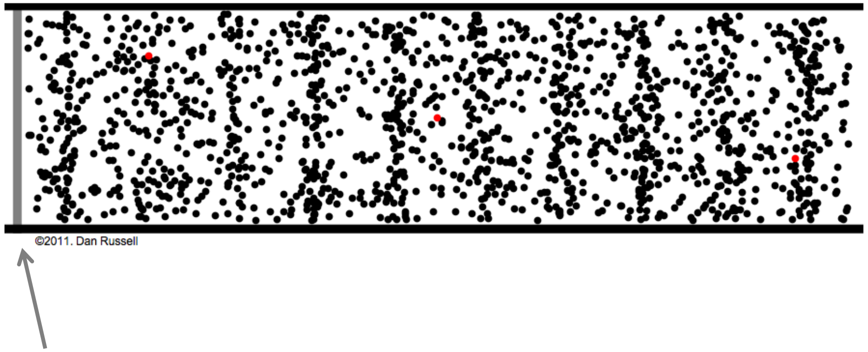
amplitude phase

$$\Delta p(t,x) = \Delta p_{max} \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$c \cdot T = \lambda, \quad c = f \cdot \lambda$

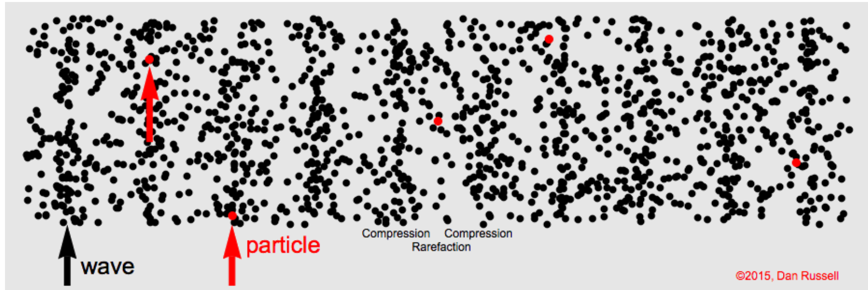
Biophysics textbook, Fig. II.46.

longitudinal wave

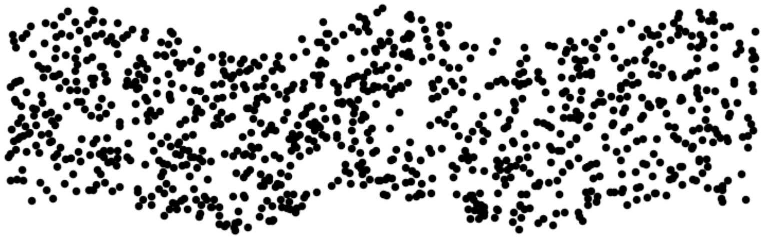


moving surface (source of wave)

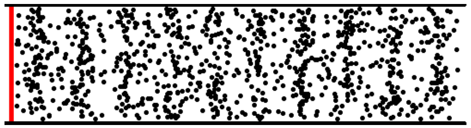
longitudinal wave



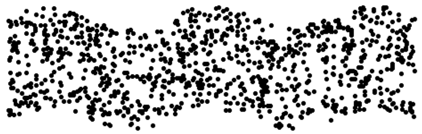
transverse wave



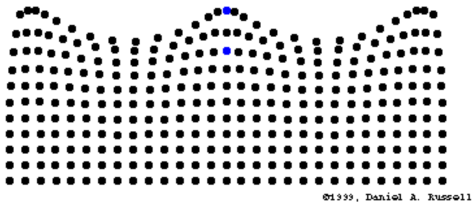
longitudinal wave



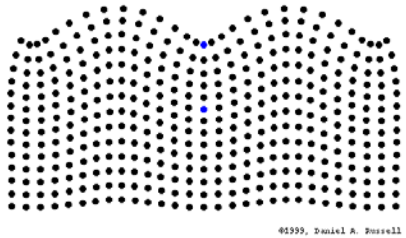
transverse wave

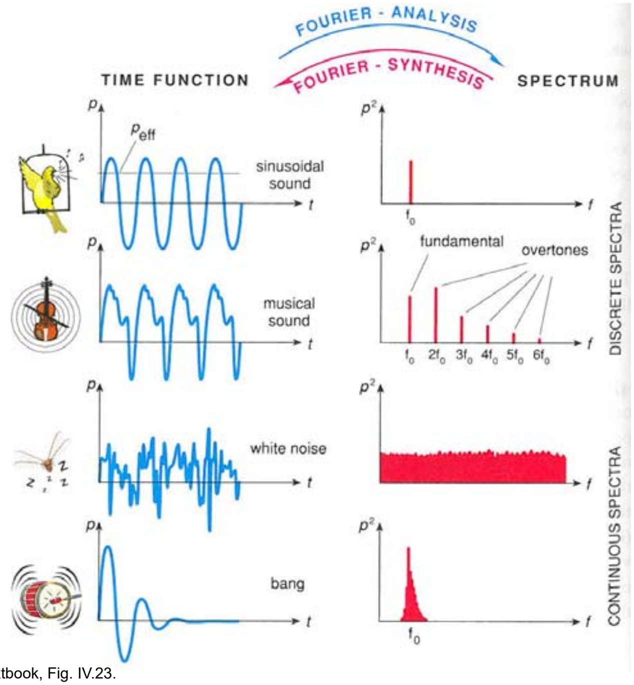


surface wave



Rayleigh wave



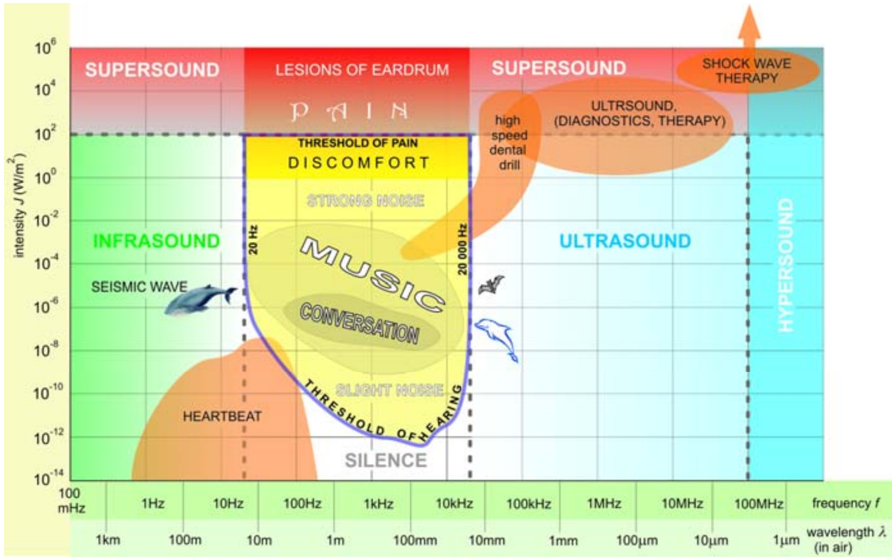


pitch:
frequency of the fundamental

timbre (tone colour):
relative strengths of overtones/harmonics (spectrum)

Textbook, Fig. IV.23.

Frequency and intensity regions of sounds



Lab. manual, Audiometry.

The role of elastic medium

$$\kappa = \frac{-\frac{\Delta V}{V}}{\Delta p}$$

compressibility
relative volume decrease over pressure

$$c = \frac{1}{\sqrt{\rho\kappa}}$$

speed of sound

$$Z = \frac{p}{v} = \frac{p_{max}}{v_{max}}$$

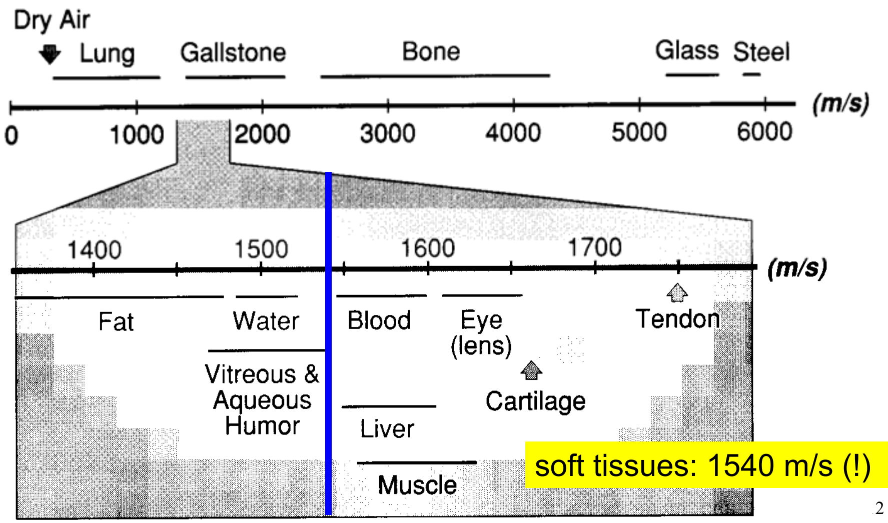
acoustic **impedance** (definition)

$$Z_{el} = \frac{U}{I}$$

$$\boxed{Z = c\rho} = \sqrt{\frac{\rho}{\kappa}}$$

acoustic **impedance** (useful form)

Speed of sound/US in different media



Intensity of US

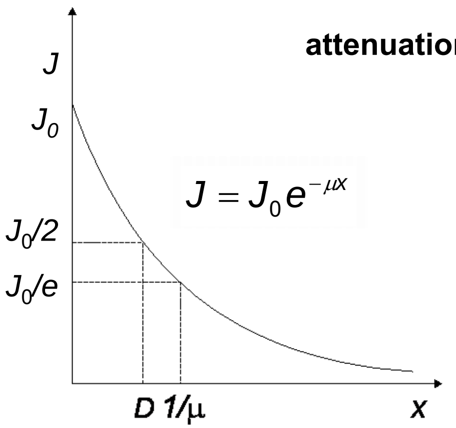
$J = \frac{1}{Z}\Delta p_{eff}^{\ 2}$

intensity = energy-current density

$P_{el} = \frac{1}{Z_{el}} U_{eff}^{\ 2}$

electric analogy

Loss of energy during propagation (absorption)



attenuation: $\alpha = 10 \cdot \lg \frac{J_0}{J}$ dB

$\alpha = 10 \cdot \mu \cdot x \cdot \lg e$ dB

μ is proportional to frequency in the diagnostic range

specific attenuation: $\frac{\alpha}{f \cdot x}$

μ is proportional to frequency in the diagnostic range

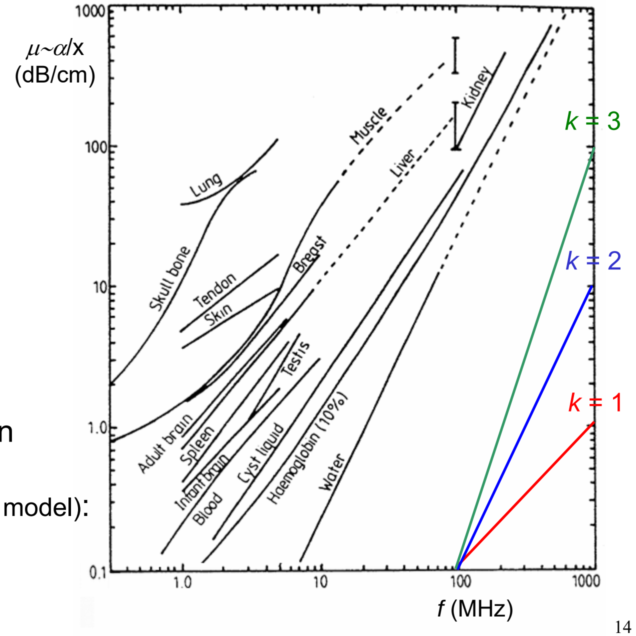
$\mu \sim f^k, \quad k \sim 1(?)$

$\log \mu \sim k \log f$

if the graph is a linear, the power function approximation is valid

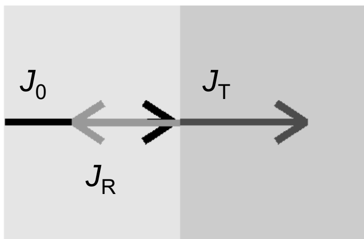
specific attenuation for soft tissues (homogeneous tissue model):

$\frac{\alpha}{f\,x} \sim 1 \frac{\text{dB}}{\text{cm MHz}}$



Phenomena at the boundary of different media

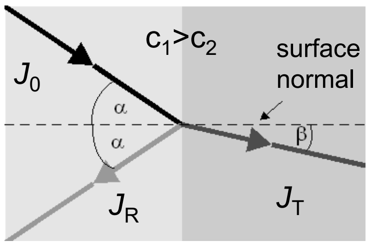
normal/perpendicular incidence



$J_0 = J_R + J_T$

reflection and transmission (penetration)

skew incidence

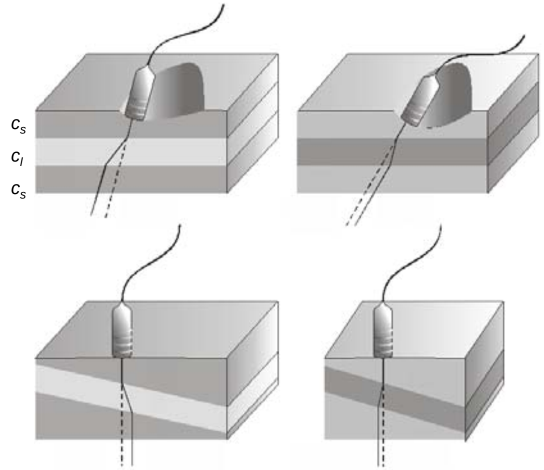


$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$

Snell's law

Textbook, Fig. II.47.

Phenomenon of skew or normal incidence and skew boundaries



position in the image and the real position are different

Textbook, Fig. on pg. 153

Reflection (normal incidence)

reflectivity:

$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

boundary surface	*R*
muscle/blood	0.001
fat/liver	0.006
fat/muscle	0.01
bone/muscle	0.41
bone/fat	0.48
soft tissue/air	0.99

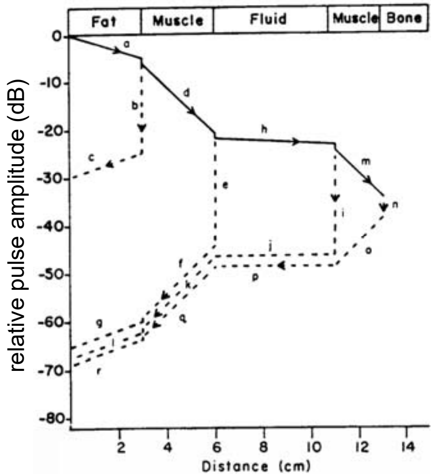
“full” reflection:

$$Z_1 << Z_2, \quad R \approx 1$$

optimal coupling:

$$Z_{\text{connecting}} \approx \sqrt{Z_{\text{source}} Z_{\text{skin}}}$$

Absorption and reflection



the later comes back the reflection, the deeper lays the reflecting surface and the weaker is the intensity

run time dependent amplification

TGC: time gain compensation

DGC: depth gain control

boundary surface	*R*	10lg*R* (dB)	*T*	10lg*T* (dB)
fat/muscle	0.01	-20.0	0.990	-0.044
muscle/blood	0.001	-30.0	0.999	-0.004
muscle/bone	0.41	-3.9	0.590	-2.291

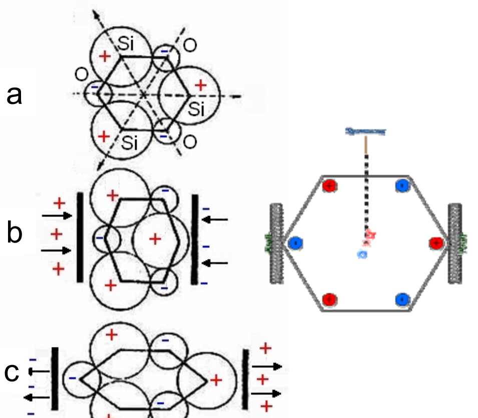
Generation of US. Piezoelectric effect

production: inverse ~
detection: direct ~

source of electric signal (sine wave oscillator)+ transducer (piezo-crystal)

(a) Center of charge of positive and negative charges coincides.

(b) and (c) As a result of pressure , the charge centers are separated, i.e. a potential difference arises (direct ~).
The crystal is deformed when voltage is applied (inverse ~).



at home:
gas igniter

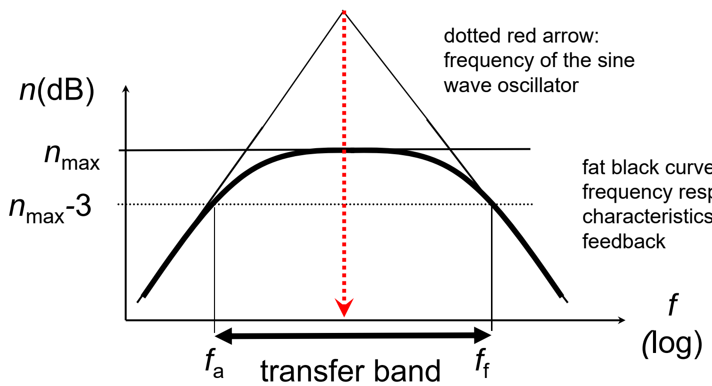
tweeter

Source of electric signal : sine wave oscillator

amplifier with positive feedback

$$A_{U,\text{feedback}} = \frac{A_U}{1 - \beta A_U}$$

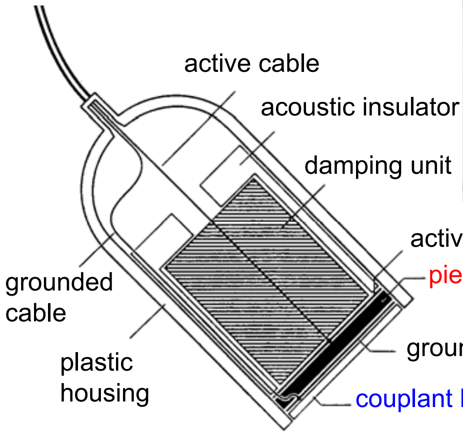
βA_U=1, amplification = „infinity“ ⇨ sine wave oscillator
no input signal, output signal: sine voltage

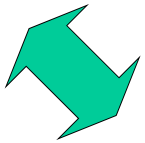


dotted red arrow: frequency of the sine wave oscillator

fat black curve: frequency response characteristics without feedback

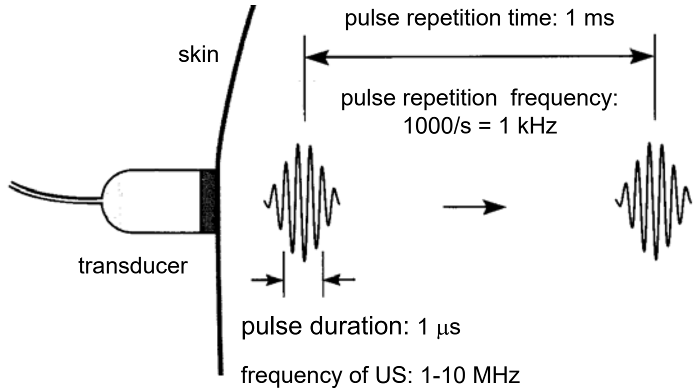
Ultrasound transducer





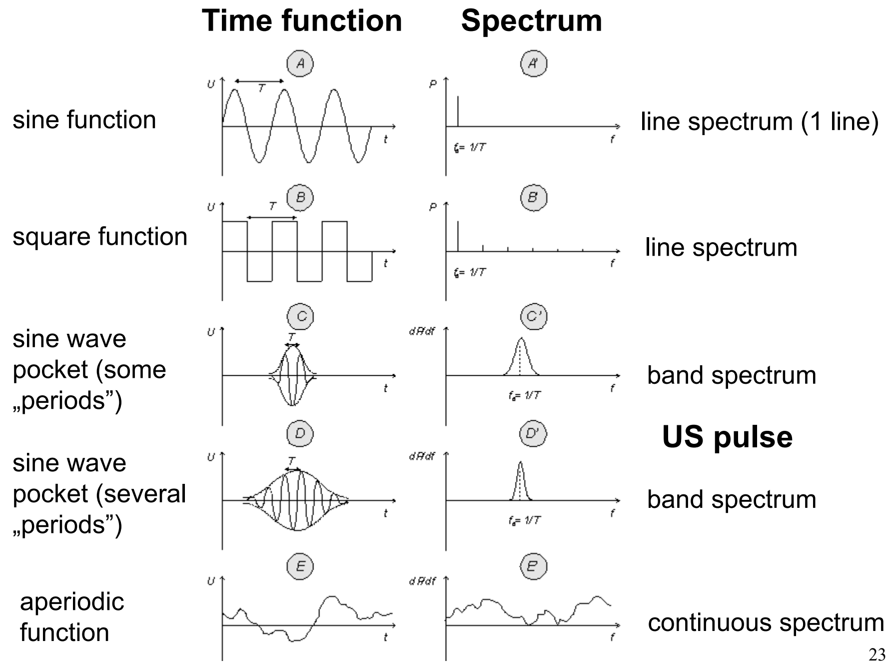
Characteristic of US pulses

transducer: transmitter and receiver is the same unit
time sharing mode: pulses instead of continuous wave US

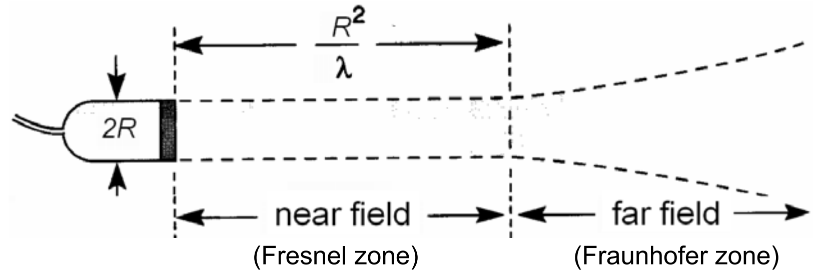


speed of propagation of US (in soft tissue): 1540 m/s

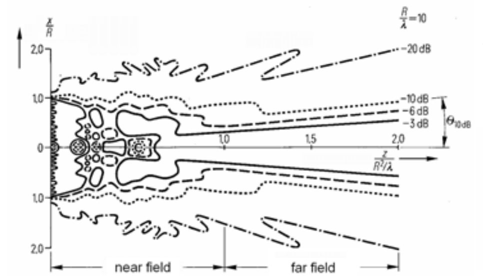
Textbook, Fig. VIII.32.



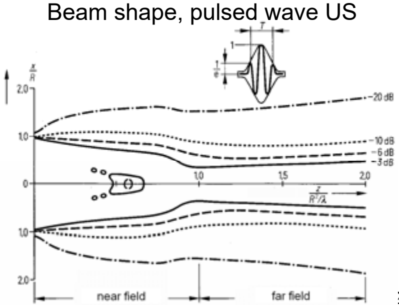
US beam shape (simplified version)

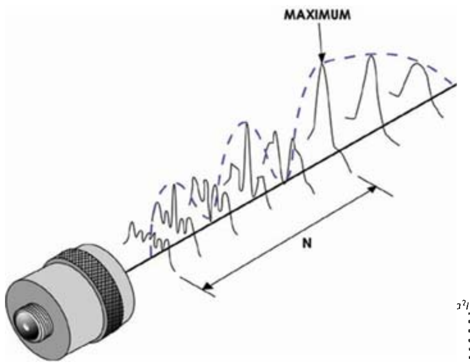


Beam shape, continuous wave US

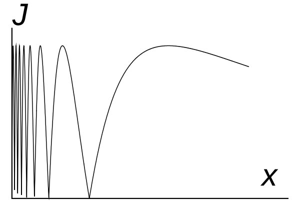


Beam shape, pulsed wave US

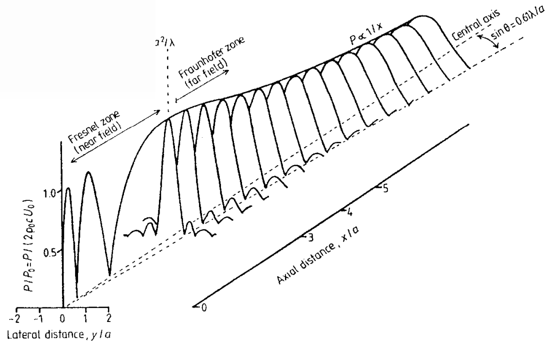




Perspective view of the US beam



change of the intensity in the axial direction



cf. Textbook. Fig. on p.505

Resolving limit, resolution

Resolving limit is the distance between two object details which can be just resolved as distinct objects (the smaller the better).
Resolution (resolving power): the reciprocal of the resolving limit (the greater the better)

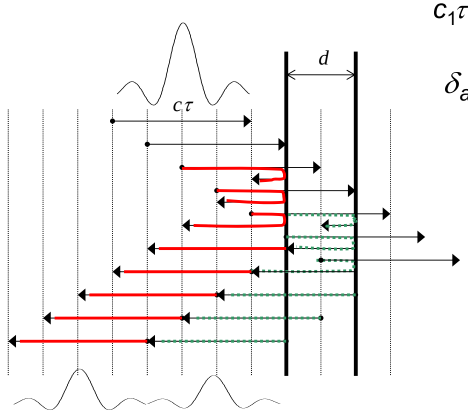
Axial resolving limit depends on the pulse length. Pulse length is inversly proportional to the frequency.

Lateral resolving limit is the minimum separation of two interfaces aligned along a direction perpendicular to the ultrasound beam. It depends on the beam width

Typical values

frequency (MHz):	2	15
wavelength (in muscle) (mm):	0.78	0.1
penetration depth (cm):	12	1.6
lateral resolving limit (mm):	3.0	0.4
axial resolving limit (mm):	0.8	0.15

Axial resolving limit



τ : pulse duration

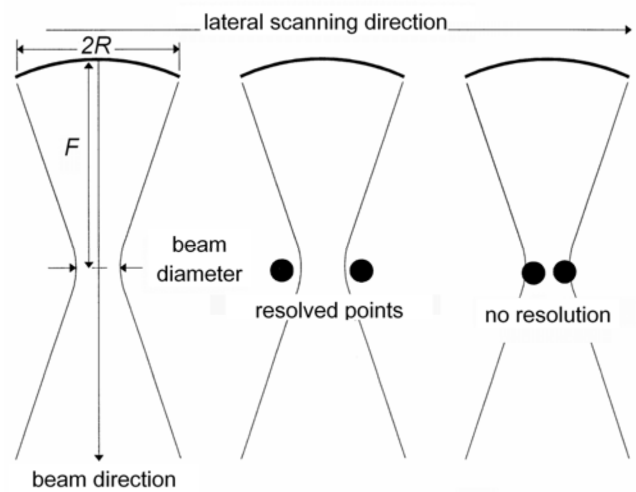
$c_1\tau \cong c_2\tau = c\tau$ pulse length

$\delta_{ax} = d = \frac{c\tau}{2}$ resolving limit

The axial resolving limit is the half of the pulse length. The echos from the adjacent surfaces in this case just hit another.

$$\tau \sim T = \frac{1}{f}$$

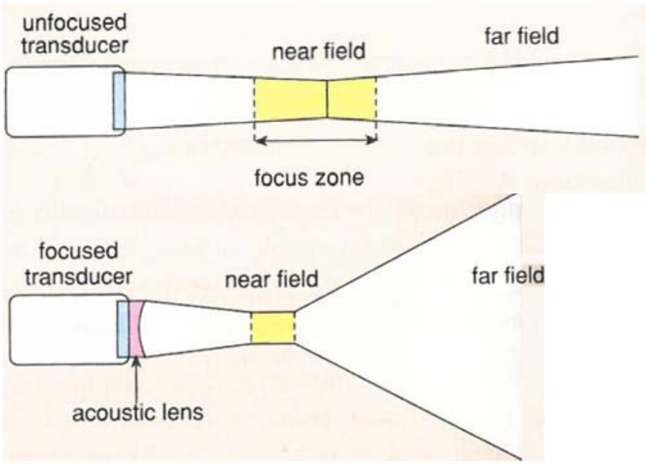
Lateral resolving limit



$$\left(\delta_{lat} \sim \frac{F}{2R} \cdot \lambda\right)$$

F: focal length
$2R$: diameter of the transducer
λ: wavelength

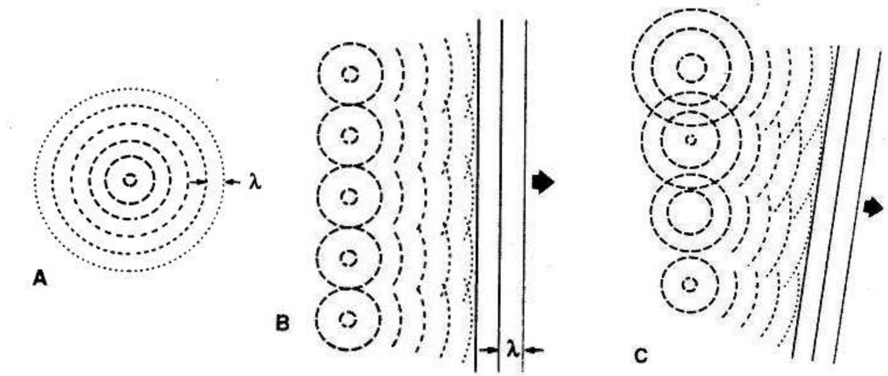
Focusing of the beam



Focusing increases the divergence of the beam in the far field regime and reduces the depth sharpness.

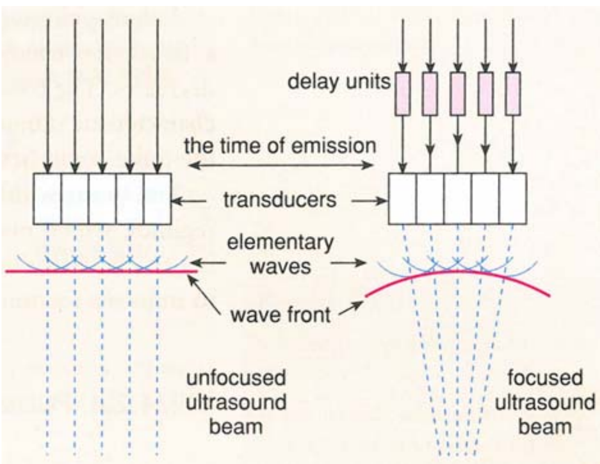
cf. Textbook Fig. on p.506

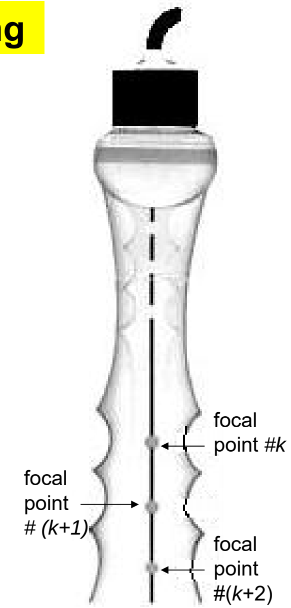
Huygens' principle



Any wave propagates so, that each point on a primary wavefront serves as the source of spherical secondary wavelets that advance with a speed and frequency equal to those of the primary wave. The primary wavefront at some later time is the envelope of these wavelets.

Electronic focusing

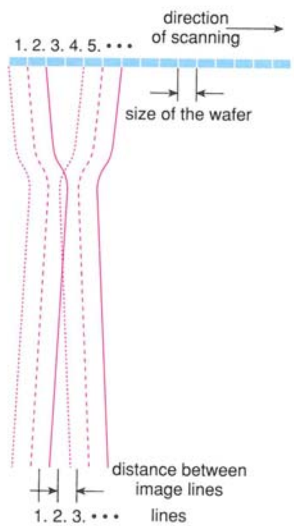




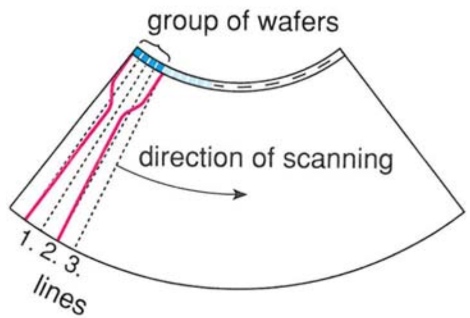
cf. Textbook Fig. on p.507

Scanning

multi unit linear array

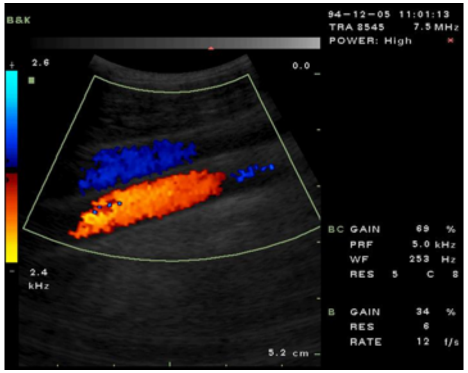


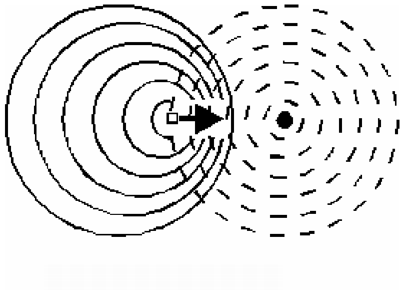
multi unit curved array



cf. Textbook Fig. VII. 36-37

US imaging. Modes of sonography. Doppler-echo.

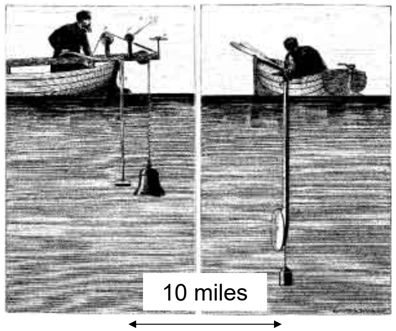


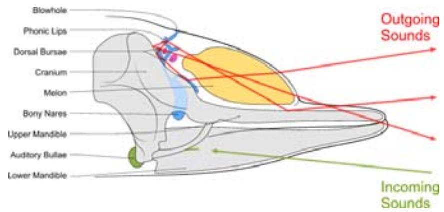


Echo principle

1794 Spallanzani:
bat's navigation

1822 Colladen
measured the speed of
sound in water

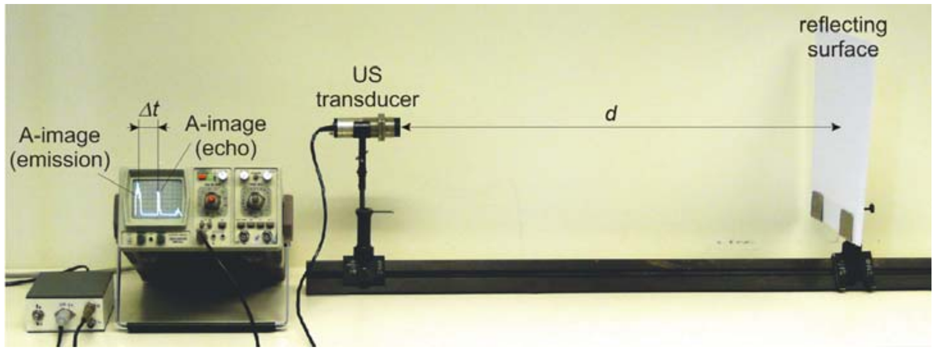




bottlenose dolphin

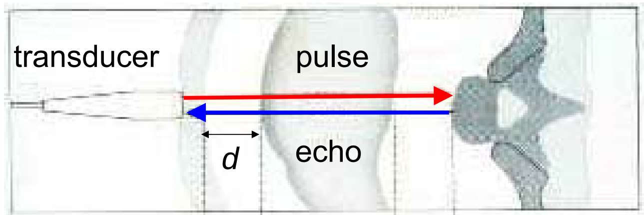
Echo principle

using a special US-head, short pulses are emitted in the air towards a reflecting surface, and the same US-head detects the echo signal

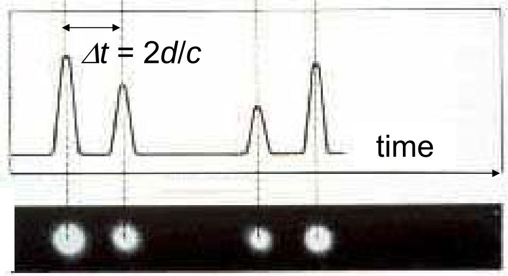


$c\Delta t = d + d = 2d$

Receiving the echos



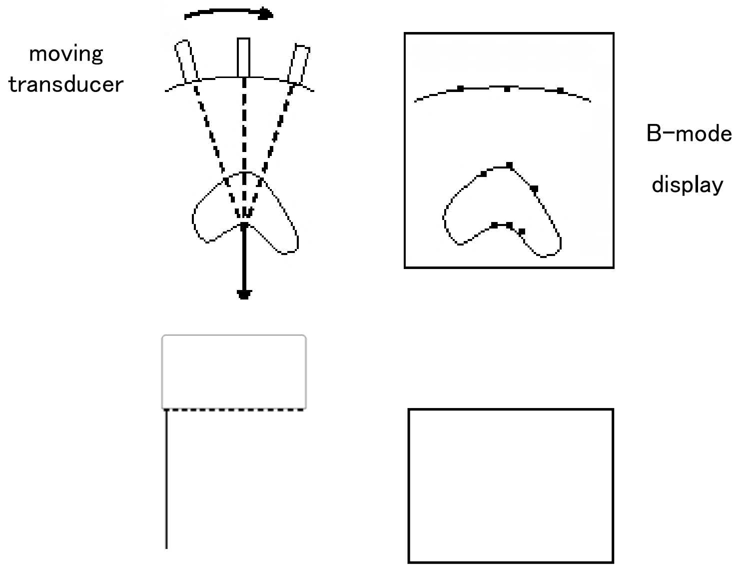
A-mode
(***A***mplitude)
only 1-dimensional



1-dimensional B-mode
(***B***rightness)

cf. Textbook Fig. VIII.33

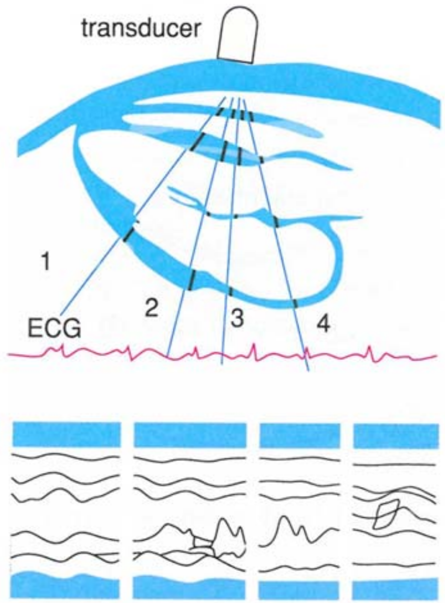
2-dimensional B-mode



TM-mode

ECG signal
for reference

(vertical)
time-dependent
1-dimensional
B-mode



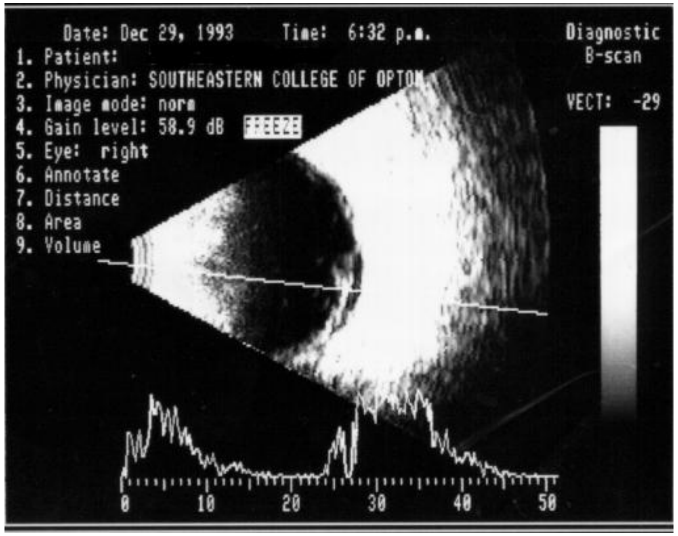
time →

(T)M-mode

Time–
Motion

Textbook Fig. VIII.34

2-dimensional B-mode and A-mode
(used in opthamology)



real speed of propagation for the accurate determination of distances:

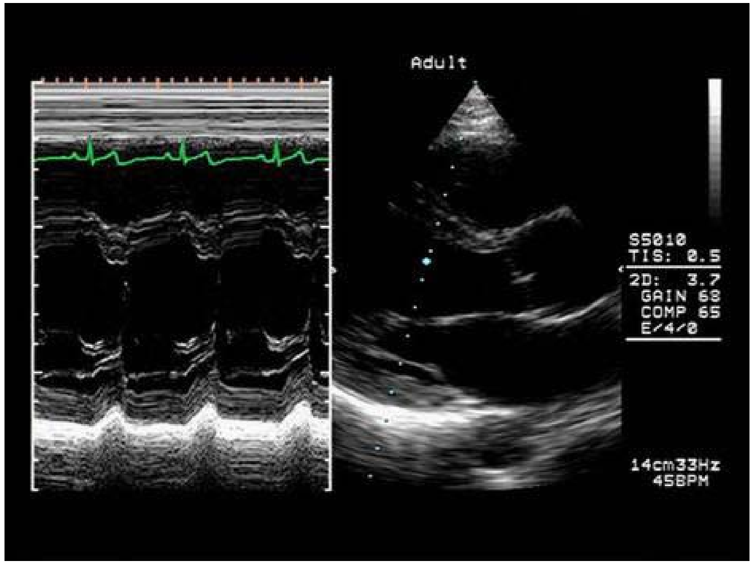
cornea: 1641 m/s

aqueos humour: 1532 m/s

crystalline lens: 1641 m/s

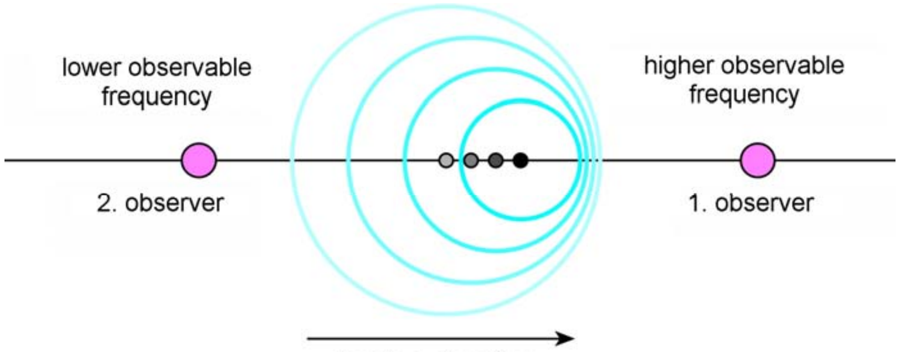
vitreous body: 1532 m/s

TM-mode B-mode



Doppler phenomenon

„The pitch of a train whistle seems to get higher as it approaches, then seems to lower as the train whistle moves away." (C. Doppler, 1842)



Teextbook Fig. VIII.39

f': **observed frequency**, f : original frequency

(a) standing source and moving observer (v_O)
+: observer approches the source
–: observer moves away from the source

$$f' = f\left(1 \pm \frac{v_O}{c}\right)$$

(b) moving source and standing observer
(if $v_S << c$, then „same" as (a))

$$f' = \frac{f}{1 \mp \frac{v_S}{c}}$$

(c) moving source and moving observer

$$f' = f\,\frac{1 \pm \frac{v_O}{c}}{1 \mp \frac{v_S}{c}}$$

(d) moving reflecting object (surface),
(if $v_R << c$)

$$f' = f\left(1 \pm \frac{2v_R}{c}\right)$$

Doppler frequency = frequency change = frequency shift

if v_i, $v_R << c$ (i= S or O)

rearranging equation (a)
moving source or observer:

$$\Delta f = f_D = \pm \frac{v_i}{c} f$$

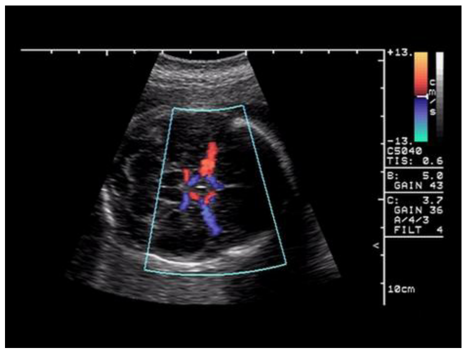
rearranging equation (d)
moving reflecting object or surface:

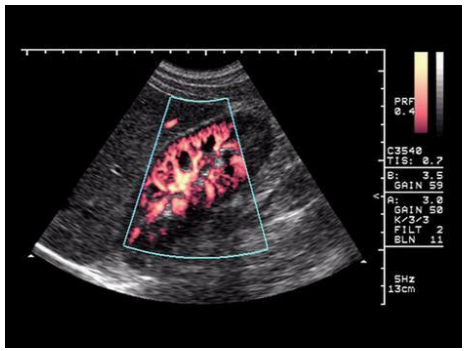
$$\Delta f = f_D = \pm 2 \frac{v_R}{c} f$$

if v and c are not parallel, then $v \cos\Theta$ should be used instead of v (remark: if $\Theta = 90°$, $f_D = 0$)

Colour coding

towards the transducer: warm colours
away from the transducer: cold colours





BART: Blue Away Red Towards

power Doppler

1-dimensional CW Doppler apparatus for measuring average flow velocity. Red blood cells as sound scatterers

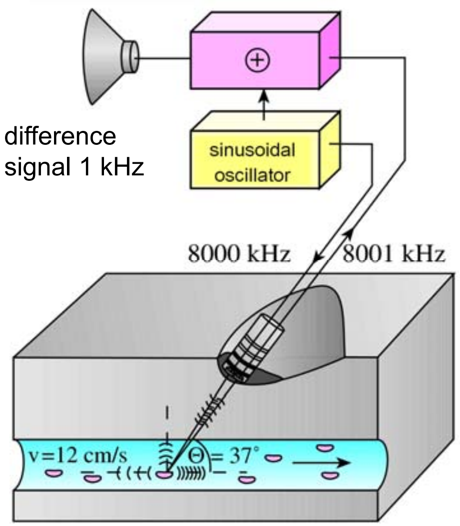
CW: continuous wave

source and detector are separated

$$|f_D| = 2\frac{v_R \cos\theta}{c} f$$

e.g. f=8000 kHz
v=12 cm/s
c=1600 m/s
Θ = 37°

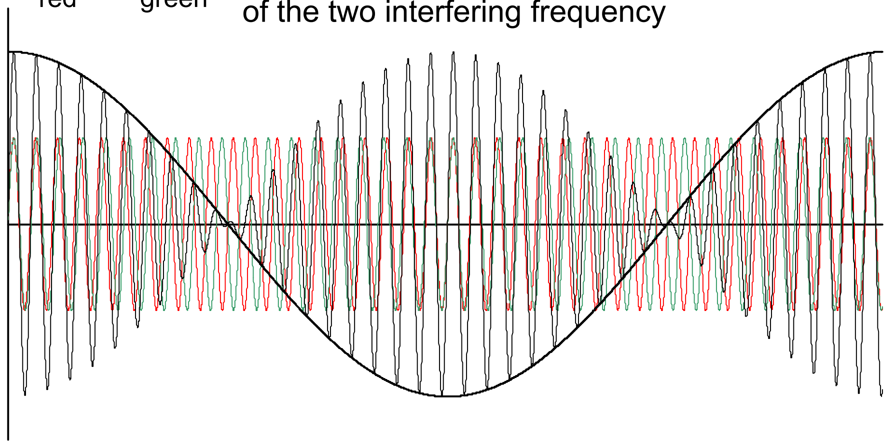
⇨f_D=1 kHz
(beating phenomenon)



Textbook Fig. VIII.41

Beating phenomenon

$f_{red} \geq f_{green}$ the beating frequency equals to the difference of the two interfering frequency

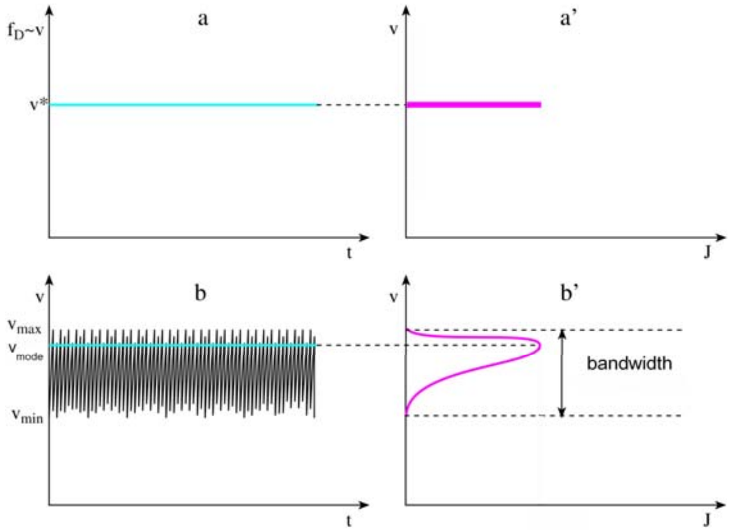


reminder: $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

Doppler curves

one constant velocity (v*)

frequency distribution (with v_{mode})



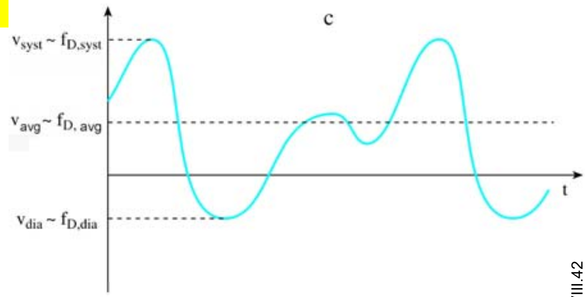
velocity distribution in TM-mode

velocity distribution at a certain time

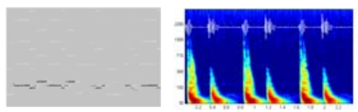
Textbook Fig. VIII.42

Doppler curves

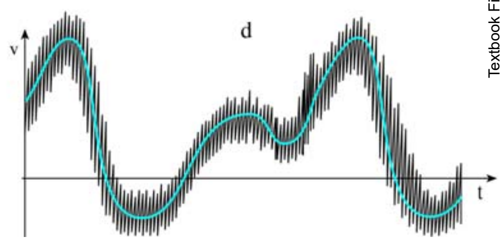
flow can be represented by one velocity in each moment



flow can be represented by a velocity distribution in each moment

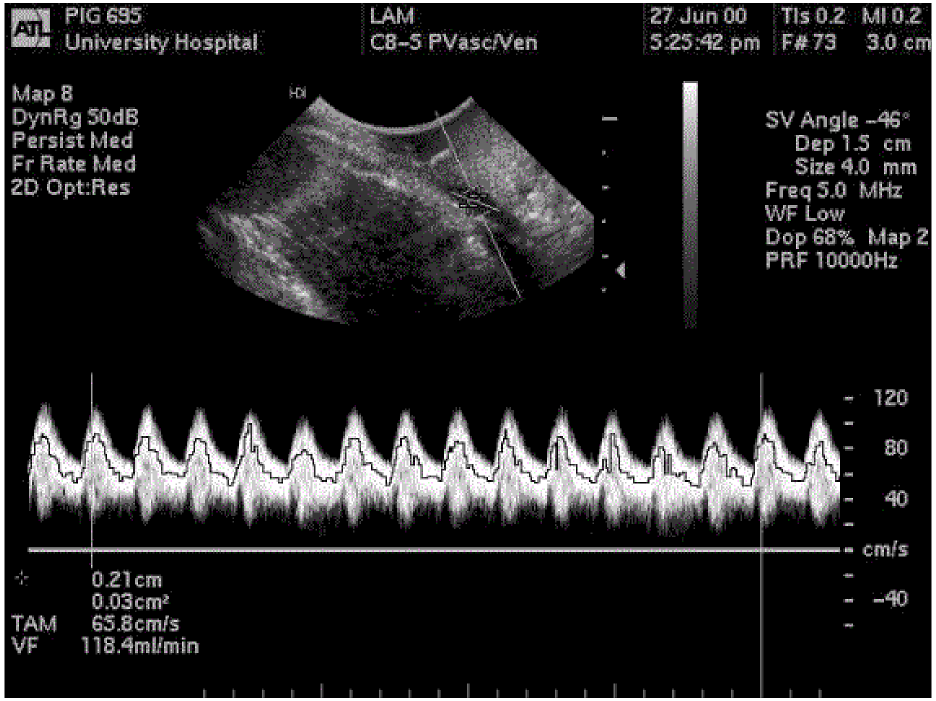


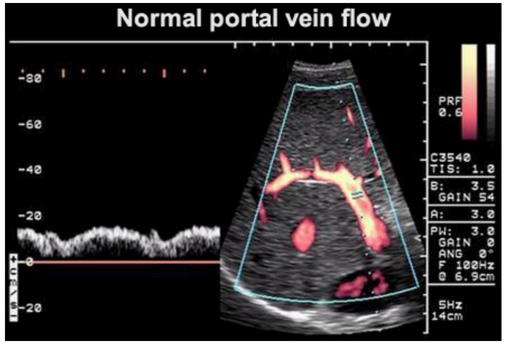
cf. voiceprint, music/heart beats in time-frequency representation

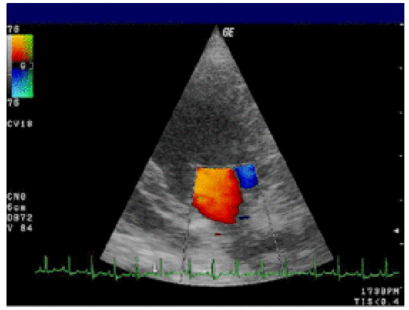


velocity distribution in TM-mode

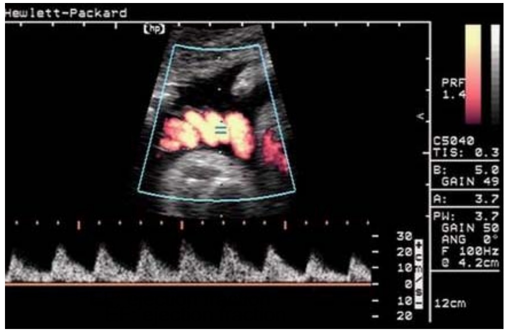
Textbook Fig. VIII.42







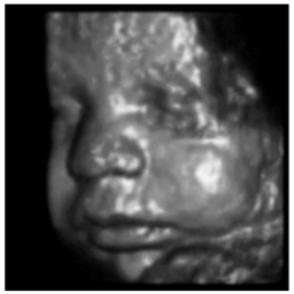
Mitral valve



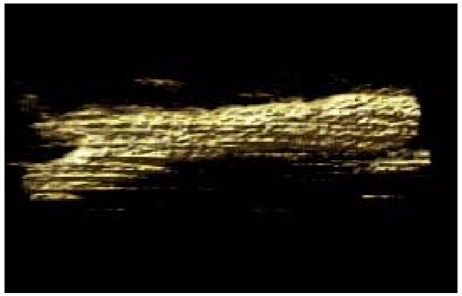
EF: ejection fraction

3D reconstruction

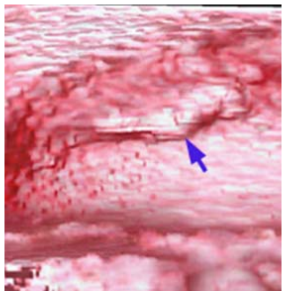
face of a fetus



carotis



bladder



1

Safety

in the diagnostics:

10 mW/cm^2 = 100 W/m^2

cf. pain threshold: 10 W/m^2

in the therapy: 1 W/cm^2

spatial average temporal average (SATA) intensity;
spatial peak temporal peak (SPTP) intensity;
spatial peak temporal average (SPTA) intensity;
spatial peak pulse average (SPPA) intensity
spatial average pulse average (SAPA) intensity