WAVE PLATES

The interaction of light with the atoms or molecules of a material is wavelength dependent. A consequence of this dependence is the resonant interactions related to material **dispersion**. Another consequence of such **resonant interaction** is **birefringence**, the change in refractive index with the polarization of light. The orderly arrangement of atoms in some crystals results in different resonant frequencies for different orientations of the electric vector relative to the crystalline axes. This, in turn, results in different refractive indices for different polarizations. Unlike dispersion, birefringence is easy to avoid: use amorphous materials such as glass, or crystals that have simple symmetries, such as NaCl or GaAs. On the other hand we can "use" birefringence to modify the polarization state of light, a useful thing to do in many situations. The optical components that do this trick are called **birefringent wave plates** or **retardation plates** (or just wave plates or retarders for short).

By taking just the right slice of a crystal with respect to the crystalline axes, we can arrange it so that the minimum index of refraction is exhibited for one polarization of the electric vector of a plane-polarized wave, as shown in Figure 1.

We say that wave is polarized along the fast axis, since its phase velocity will be a maximum. A plane-polarized wave with its plane rotated 90° will propagate with the maximum index of refraction and minimum phase velocity, as shown in Figure 1.

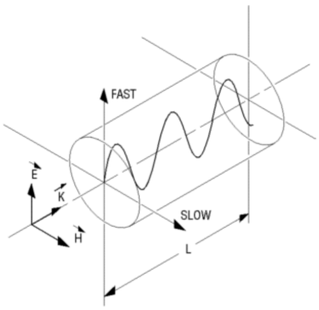

Fig. 1.

We say it is polarized along the slow axis. The difference in the number of wavelengths shown in Figures 1 and 2 (2 2/3, and 4 respectively) would imply a ratio of the two indices of refraction nfast/nslow = 2/3, a much larger difference than in typical natural crystals; we have exaggerated the ratio for clarity.

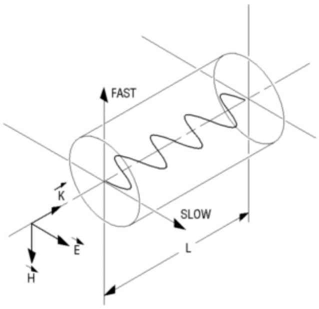

Fig. 2.

The propagation phase constant k can be written as 2pfn/c radians per meter, so that a wave of frequency f will experience a phase shift of ø = 2pfnL/c radians in travelling a distance L through the crystal. Thus, the phase shift for the wave in Figure 1 will be øfast = 2pfnfastL/c, and for the wave in Figure 2, øslow = 2pnslowfL/c (8p radians as shown.) The difference between these two phase shifts is termed the **retardation** G = 2pf(nslow -nfast)L/c. The value of G in this formula is in radians, but is more common to express in "wavelengths" or "waves", with a "full wave" meaning G = 2p, a "half-wave" meaning G = p, a "quarter-wave" meaning G = p/2, and so forth. Thus, we would term the crystal shown in the Figures a "4/3 wave plate"; that is, it retards the phase of the slow wave by 4/3 of a wave (cycle) relative to the fast wave.

Since waves repeat themselves every 2p radians, we could just as well subtract out an integral number of 2ps or waves and call the crystal shown a 2p/3 radian or 13 wave plate. We would never know the difference, provided we only used it at exactly the optical frequency shown in the Figures. However, if we change the frequency we will quickly note that the retardation will change at a rate faster than it would for a plate that had really only 13 wave retardation. We can note this difference by calling it a "**multiple order** 13 wave plate."

Half-wave Plates

By far the most commonly used wave plates are the half-wave plate (G = p) and the quarter-wave plate (G = p/2). The half-wave plate can be used to rotate the plane of plane polarized light as shown in Figure 3.

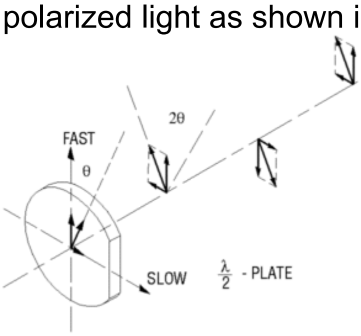

Fig. 3.

Suppose a plane-polarized wave is normally incident on a wave plate, and the plane of polarization is at an angle q with respect to the fast axis. To see what happens, resolve the incident field into components polarized along the fast and slow axes, as shown. After passing through the plate, pick a point in the wave where the fast component passes through a maximum. Since the slow component is retarded by one half-wave, it will also be a maximum, but 180° out of phase, or pointing along the negative slow axis. If we follow the wave further, we see that the slow component remains exactly 180° out of phase with the original slow component, relative to the fast component. This describes a plane-polarized wave, but making an angle q on the opposite side of the fast axis. Our original plane wave has been rotated through an angle 2q. You can satisfy yourself that you will find the same result if the incident wave makes an angle q with respect to the slow axis.

A half-wave plate is very handy in rotating the plane of polarization from a polarized laser to any other desired plane (especially if the laser is too large to rotate). Most large ion lasers are vertically polarized, for example, so to obtain horizontal polarization, simply place a half-wave plate in the beam with its fast (or slow) axis 45° to the vertical. If it happens that your half-wave plate does not have marked axes (or if the markings are obscured by the mount), put a polarizer in the beam first and orient it for extinction (horizontally polarized), then interpose the half-wave plate normal to the beam and rotate it around the beam axis so that the beam remains extinct, you have found one of the axes. Then rotate the half-wave plate exactly 45° around the beam axis (in either direction) from this position, and you will have rotated the polarization of the beam by 90°. You may check this by rotating the polarizer 90° to see that extinction occurs again. If you need some other angle, instead of 90° polarization rotation, simply rotate the wave plate by half the angle you desire. A convenient wave plate mount calibrated in angle is the **RSP-1T** (section 6).

Incidentally, if the polarizer doesn't give you as good an extinction as you had before you inserted the wave plate, it likely means your wave-plate isn't exactly a half-wave plate at your operating wavelength. You can correct for small errors in retardation by rotating the wave plate a small amount around its fast or slow axes. Rotation around the fast axis decreases the retardation while rotation around the slow axis increases the retardation. Try it both ways and use your polarizer to check for improvement in extinction ratio.

Quarter-wave Plates

Quarter-wave plates are used to turn plane-polarized light into circularly-polarized light and vice versa. To do this, we must orient the wave plate so that equal amounts of fast and slow waves are excited. We may do this by orienting an incident plane-polarized wave at 45° to the fast (or slow) axis, as shown in Figure 4.

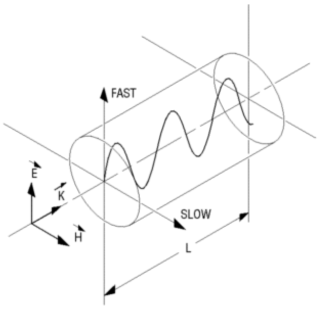

Fig. 4.

On the other side of the plate, we again examine the wave at a point where the fast-polarized component is maximum. At this point, the slow-polarized component will be passing through zero, since it has been retarded by a quarter-wave or 90° in phase. If we move an eighth wavelength farther, we will note that the two are the same magnitude, but the fast component is decreasing and the slow component is increasing. Moving another eighth wave, we find the slow component is maximum and the fast component is zero. If we trace the tip of the total electric vector, we find it traces out a **helix**, with a period of just one wavelength. This describes **circularly polarized light**. Right-hand light is shown in the Figure; the helix wraps in the opposite sense for left-hand polarized light. You may produce left-hand polarized light by rotating either the wave plate or the plane of polarization of the incident light 90° in the Figure.

Setting up a wave plate to produce circularly polarized light proceeds exactly as we described for rotating 90° with a half-wave plate: first, cross a polarizer in the beam to find the plane of polarization. Next, insert the quarter-wave plate between the source and the polarizer and rotate the wave plate around the beam axis to find the orientation that **retains**the extinction. Then rotate the wave-plate 45° from this position. You should now have half the incident light passing through the polarizer (the other half being absorbed or deflected, depending on which kind of polarizer you are using). You can check the quality of the circularly polarized light by rotating the polarizer -- the intensity of light passing through the polarizer should remain unchanged. If it varies somewhat, it means the light is actually **elliptically polarized**, and your wave plate isn't exactly a quarter-wave plate at your operating wavelength. You may correct this as with the half-wave plate by tilting the wave-plate about its fast or slow axes slightly, while rotating the polarizer to check for constancy.

You may wonder what effect retardations other than a half-wave or a quarter-wave have on linearly polarized light. Figure 5 shows the effect of retardation on plane polarized light with the plane of polarization making an arbitrary angle with respect to the fast axis.

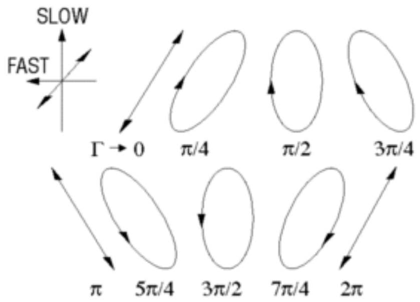

Fig. 5.

The result is elliptically polarized light, with the amount of ellipticity and the tilt of the axis of the ellipse a function of the retardation and the tilt of the incident plane wave. The exception is a half-wave retardation, in which case the ellipse degenerates into a plane wave making an angle of 2q with the fast axis. Note that the quarter-wave plate does not produce circularly polarized light here, because equal amounts of fast and slow wave components were not used; the incident tilt angle must be exactly 45° with respect to the fast (or slow) axis to make these components equal.

Wave Plate Applications

We have already mentioned the two most common applications of wave plates: rotating the plane of polarization with a half-wave plate and creating circular polarization with a quarter-wave plate. Obviously, you can also use a quarter-wave plate to create plane polarization from circular polar-ization -- just reverse the direction of light propagation in Figure 4.

Optical Isolation --

We can use a quarter-wave plate as an optical isolator, that is, a device that eliminates undesired reflections. Such a device uses a quarter-wave plate and a polarizing beamsplitter cube. The diagram on page 1. 20 shows how to construct an isolator in this manner.

Polarization Cleanup --

Often an optical system will require several reflections from metal or dielectric mirrors. There is no change in the polarization state of the reflection if the beam is incident normally on the mirrors, or if the plane of polarization lies in or normal to the plane of incidence. However, if the polarization direction makes some angle with the plane of incidence, then the reflection often makes a small phase shift between the parallel and perpendicular components. This is particularly true for metal mirrors, which always have some loss. The resulting reflected wave is no longer plane polarized, but will be slightly elliptically polarized, as you can easily determine by its degraded extinction when you insert a polarizer and rotate it. This small ellipticity can often be removed by inserting a **full wave plate** (which ordinarily does nothing) and tilting it slightly about either fast or slow axes to change the retardation slightly to just cancel the ellipticity.

Wave Plate Material and Practice

Materials --

Many natural occurring crystals exhibit birefringence, and could, in principle, be used for wave plates. Calcite and crystalline quartz are typical materials. They are durable and of high optical quality. However, the refractive index difference, nslow - nfast is so large that a true half-wave plate would be impracticably thin to polish.

It is also possible to induce small amounts of birefringence into a normally isotropic material through stress. For example, most plastics exhibit birefringence from stress applied in the manufacture. Plastic wave plate material is available in half- or quarter-wave retardation values in very large sheets. It is inexpensive, but not of the highest optical quality or durability.

Multiple-order wave plates --

One alternative to polishing or cleaving very thin plates is to use a practical thickness of a durable material such as crystalline quartz and obtain a high-order wave plate, say a 15.5 wave plate for a 1 mm thickness. Such a plate will behave exactly the same as a half-wave plate at the design wavelength. However, as the optical wavelength is changed, the retardation will change much more rapidly than it would for a true half-wave plate. The formula for this change is easily derived from the definition of G:

$$\Gamma = (2m+1)\pi\left(\frac{\delta f}{f_0}\right)$$

$$\approx -(2m+1)\pi\left(\frac{\delta\lambda}{\lambda_0}\right)$$

where f0 and l0 are the design frequency and wavelength, and m is the order of the wave plate. Thus, the rate of change of retardation with frequency dG/df will be 2m + 1 times as large for an mth order plate as a true half-wave plate, (m = 0, or "zero order" plate). This would be 31 times larger for our 1 mm "15.5-wave" plate! You should calculate the frequency or wavelength range your system requires, and see if the error in retardation will be tolerable over that range with a multiple order wave plate.

By like reasoning, the sensitivity of the retardation to rotation about the fast and slow axes is found to be about (2m + 1) times larger for a multiple order plate than a true zero-order half-wave plate. This means much smaller rotations are required to correct for retardation errors. But it also means that light rays not parallel to the optical axis will see a (2m + 1) larger change in retardation. Multiple order wave plates are not recommended in strongly converging or diverging beam portions of your optical system. Similarly, the sensitivity of retardation to changes in length caused by changes in temperature are multiplied by (2m + 1), so that tighter temperature control will be required. A typical temperature sensitivity is 0.0015 wave per degree C for a visible 1 mm thick half-wave plate.

Multiple-order wave plates can be used to advantage if you require a wave plate that can be used at two discrete wavelengths, for example the 488 and 514 nm wavelengths of an argon-ion laser or the 532 and 1064 nm wavelengths from a Nd:YAG laser. By choosing the thickness to give a (2m1 + 1) plate at one wavelength and a (2m2 + 1) plate at the other, both wavelengths will see a "half-wave" plate (but not the wavelengths in between)! The integers have to be chosen by a computer program, since the dispersion in index has to be accounted for also, but it is usually possible to find a plate of reasonable thickness provided the two wavelengths are not too close together.

Zero-order wave plates --

Fortunately, a technique is available for realizing true half-wave plate performance, while retaining the high optical quality and rugged construction of crystalline quartz wave plates. By combining two wave plates whose retardations differ by exactly half a wave, a true half-wave plate results. The fast axis of one plate is aligned with the slow axis of the other, so that the net retardation is the difference of the two retardations. The change in retardation with frequency (or wave-length) is minimized. Temperature sensitivity is also reduced; a typical value is 0.0001-wave per degree C. The change in retardation with rotation is highly dependent on manufacturing conditions and may be equal to greater than that of a multiple order wave plate.

These wave plates are recommended for use in systems using tunable radiation sources, such as a dye laser or white light sources.